2008 TI Cup Mathematics Contest

(Grade 11)

Script for Individual Contest

(9:00AM ~ 10:30AM, May 25, 2008)

Admission No._____ Gender_____ Grade_____ Name_____ School_____

Binding

No.	I	II	III	IV	Total Score
Score					
Marked by					
Reviewed by					

I. Fill in the blanks: (total 60 marks for 8 questions, with 6 marks for each of the first 4 questions and 9 marks for each of the last 4 questions)

1. The positive real solution(s) of the equation $5^5+x^5+7^5+x^5+8^5=54748$ is/are ____________.

2. Evaluate:

   (1) $\sqrt{627953481}+\sqrt{672935481}=$ __________________;

   (2) $\sqrt{\sqrt{254817369}-\sqrt{152843769}}=$ ________________.

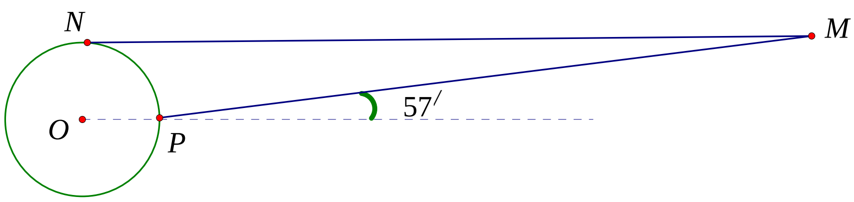
3. The cubic function $y=ax^3+bx^2+cx+d$ that passes through the points (3,8), (4,30), (5,72) and (6,140) is _____________________.

4. If the numbers shown on die A and B are a and b respectively, then there are _________ pairs of (a, b) such that the quadratic equation $x^2-2(a-3)x-b^2+9=0$ has real solutions.

5. Find all the 3-digit positive integers that are equal to the last 3 digits of their respective squares: _____________________.

6. The maximum value of the function $y=\sqrt{3-4x}+\sqrt{2x+1}$ is _____________.

7. The moon M is right on the horizon when it is observed from the North Pole N, and at 57' north to the zenith when it is observed from a point P on the equator. Given that the radius of the earth R is 6370 km, and the center of earth O lies on the same plane containing N, P and M, the minimum distance from the moon M to the surface of the earth is ______________km (to the nearest 10^3 km).



8. Define $s(m)$ as the sum of all digits of the positive integer m, then $s(1)+s(2)+\cdots+s(2008)=$_______.

Please give the necessary steps in your solutions to the following 3 problems:

II. (20 marks) Find all positive integers n such that

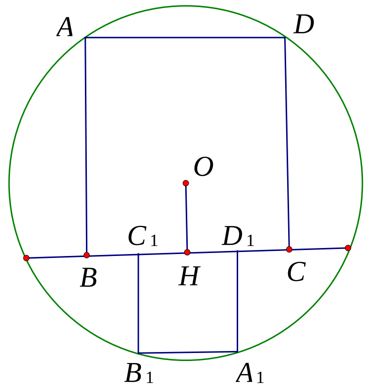
$$\left[\frac{n}{2}\right]+\left[\frac{n}{3}\right]+\left[\frac{n}{4}\right]+\left[\frac{n}{5}\right]=69,$$

where $[x]$ denotes the greatest integer less than or equal to x.

[Solution]

III. (20 marks) As shown in the figure, OH, the perpendicular distance from the center of a circle O to a chord, is 2008. Two squares $ABCD$ and $A_1B_1C_1D_1$ are constructed inside the two segments of the circle, such that the points A, D, A_1, B_1 are on the circumference, and the points B, C, C_1, D_1 are on the chord. Find the difference between the lengths of the sides of the two squares.

[Solution]



IV. (20 marks) A disk G (including its circumference) is centered at the point $A(2008, 525)$ with radius 1 in the Cartesian plane. Given that the parabola $y^2 = 2px$ intersects disk G, find the range of possible values of p (correct to the nearest 0.001).

[Solution]