VCE General Mathematics

2024 Examination 2 Solutions

Solutions by:

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Write your **student number** in the boxes above.

Letter

General Mathematics Examination 2

Question and Answer Book

VCE Examination – Monday 4 November 2024

- Reading time is **15 minutes**: 2.00 pm to 2.15 pm
- Writing time is **1 hour 30 minutes**: 2.15 pm to 3.45 pm

Approved materials

- One bound reference that may be annotated
- One approved CAS calculator or CAS software, and one scientific calculator

Materials supplied

- Question and Answer Book of 28 pages
- Formula Sheet

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents   pages

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In all questions where a numerical answer is required, you should only round your answer when instructed to do so.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data analysis

Question 1 (8 marks)

Table 1 lists the Olympic year, *year*, and the gold medal-winning height for the men's high jump, *Mgold*, in metres, for each Olympic Games held from 1928 to 2020. No Olympic Games were held in 1940 or 1944, and the 2020 Olympic Games were held in 2021.

Table 1

year	Mgold (m)
1928	1.94
1932	1.97
1936	2.03
1948	1.98
1952	2.04
1956	2.12
1960	2.16
1964	2.18
1968	2.24
1972	2.23
1976	2.25
1980	2.36
1984	2.35
1988	2.38
1992	2.34
1996	2.39
2000	2.35
2004	2.36
2008	2.36
2012	2.33
2016	2.38
2020	2.37

	A year	B mgold
=		
1	1928	1.94
2	1932	1.97
3	1936	2.03
4	1948	1.98
5	1952	2.04
B1	1.94	

Data: <www.olympics.com/en/olympic-games/olympic-results>

a. For the data in Table 1, determine:

    i. the maximum *Mgold* in metres   1 mark

2.39 m

    ii. the percentage of *Mgold* values greater than 2.25 m.   1 mark
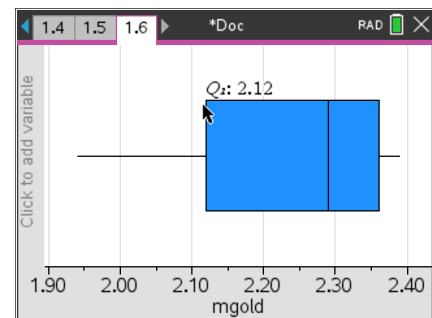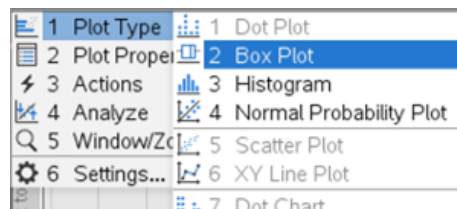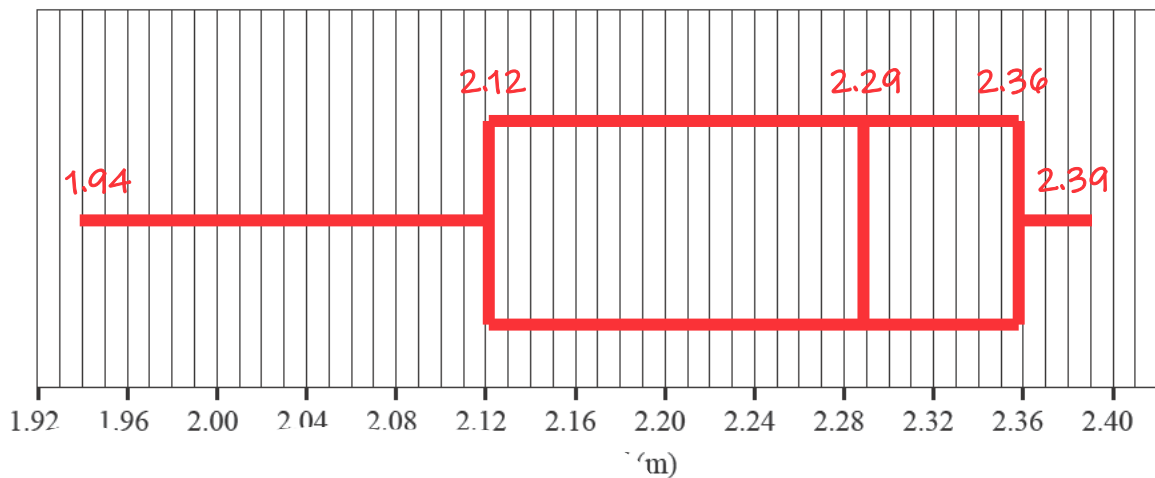
50%

b. The mean of these *Mgold* values is 2.23 m, and the standard deviation is 0.15 m.

Calculate the standardised z-score for the 2000 *Mgold* of 2.35 m.   1 mark

z-score = $\dfrac{2.35 - 2.23}{0.15}$

= 0.8

c. Construct a boxplot for the *Mgold* data in Table 1 on the grid below.   2 marks

Question 1 continues on the next page.

d. A least squares line can also be used to model the association between *Mgold* and *year*.

Using the data from Table 1, determine the equation of the least squares line for this data set.

Use the template below to write your answer.

Round the values of the intercept and slope to ~~three significant figures.~~   **2 marks**

$$Mgold = \boxed{-7.97} + \boxed{0.00516} \times year$$

e. The coefficient of determination is 0.857

Interpret the coefficient of determination in terms of *Mgold* and *year*.   **1 mark**

_____

85.7% of variation in Mgold can be explained by the variation in year

_____

_____

_____

1.3	1.4	1.5 ▶	*Doc		RAD ▯ ✕
	B mgold	C	D	E	=LinRegB
=				=LinRegB	
1	1.94		Title	Linear R...	
2	1.97		RegEqn	a+b*x	
3	2.03		a	-7.9710...	
4	1.98		b	0.00516..	
5	2.04		r²	0.85743..	
E1	="Linear Regression (a+bx)"			◀ ▶	

Question 2 (4 marks)

The boxplot below displays the distribution of all gold medal-winning heights for the women's high jump, $Wgold$, in metres, for the 19 Olympic Games held from 1948 to 2020.
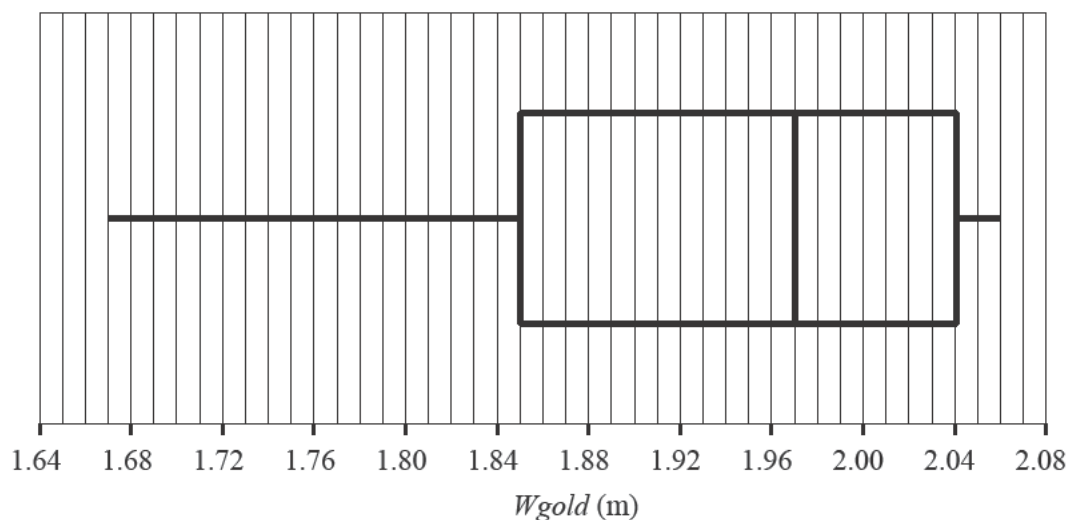
$Wgold$ (m)

a. Describe the shape of this data distribution.   1 mark

<u>   Negatively Skewed   </u>

b. For this boxplot, what is the **smallest possible** number of $Wgold$ heights lower than 1.85 m?   1 mark

Q1 =1.85.   Therefore only need 1 value to extend the whisker.

c. i. Using the boxplot, show that the lower fence is 1.565 m and the upper fence is 2.325 m.   1 mark

IQR = 2.04-1.85 = 0.19

lower fence = 1.85 - 1.5 x 0.19 = 1.565

Upper fence = 2.04 + 1.5 x 0.19 = 2.325

ii. Referring to the boxplot, the lower fence and the upper fence, explain why no outliers exist.   1 mark

the minumum value on the boxplot is 1.67 which is above the lower fence (1.565).

The maximum value on the boxplot is 2.06 which is belowthe upper fence (2.325).
Therefore no outliers exist.

Question 3 (10 marks)

The Olympic gold medal-winning height for the women's high jump, *Wgold,* is often lower than the best height achieved in other international women's high jump competitions in that same year.

Table 2 lists the Olympic year, *year*, the gold medal-winning height, *Wgold*, in metres, and the best height achieved in all international women's high jump competitions in that same year, *Wbest*, in metres, for each Olympic year from 1972 to 2020.

A scatterplot of *Wbest* versus *Wgold* for this data is also provided.

Table 2

year	1972	1976	1980	1984	1988	1992	1996	2000	2004	2008	2012	2016	2020
Wgold (m)	1.92	1.93	1.97	2.02	2.03	2.02	2.05	2.01	2.06	2.05	2.05	1.97	2.04
Wbest (m)	1.94	1.96	1.98	2.07	2.07	2.05	2.05	2.02	2.06	2.06	2.05	2.01	2.05
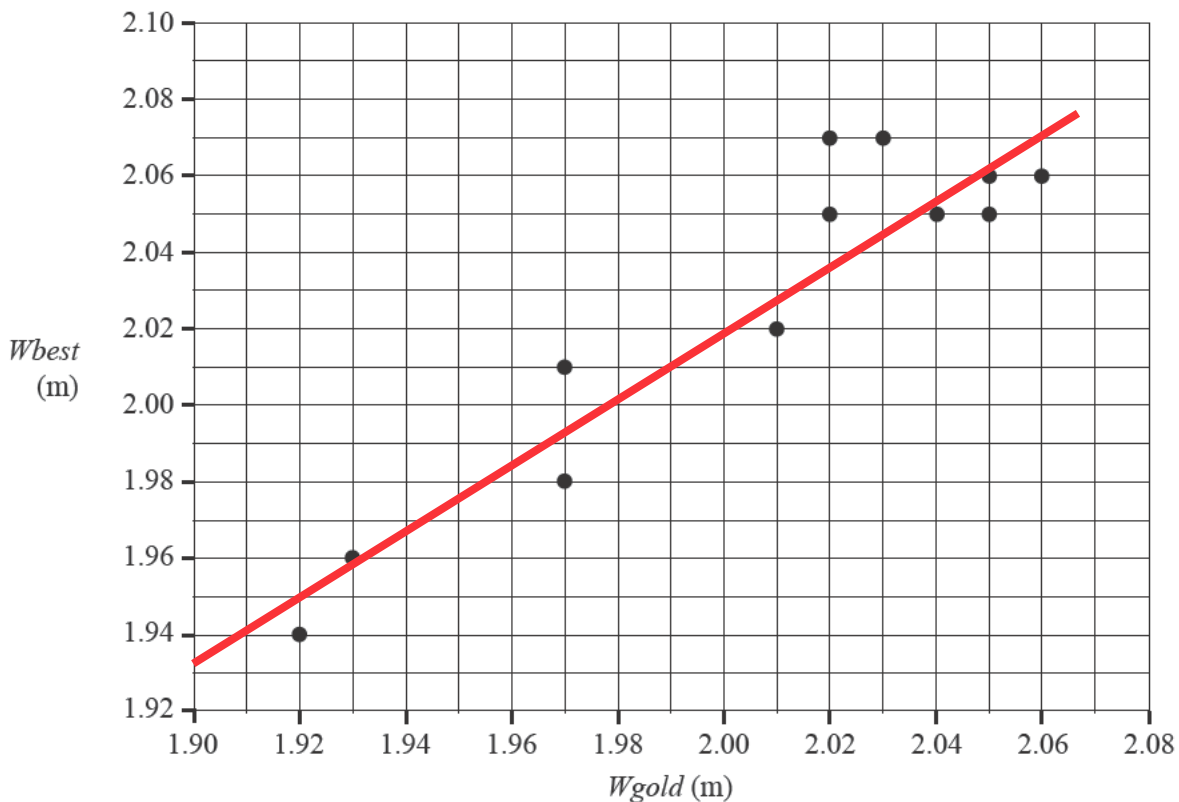
calculate 2 points

$0.3 + 0.86 \cdot x | x = 1.9$ 1.934

$0.3 + 0.86 \cdot x | x = 2.06$ 2.0716

When a least squares line is fitted to the scatterplot, the equation is found to be:

$$Wbest = 0.300 + 0.860 \times Wgold$$

The correlation coefficient is 0.9318

a. Name the response variable in this equation.   1 mark

Wbest

b. Draw the least squares line on the scatterplot on **page 6**.   1 mark

c. Determine the value of the coefficient of determination as a percentage. Round your answer to one decimal place.

86.8%

d. Describe the association between *Wbest* and *Wgold* in terms of strength and direction.   1 mark

strength	strong
direction	positive

e. Referring to the equation of the least squares line, interpret the value of the slope in terms of the variables *Wbest* and *Wgold*.   1 mark

On average, for each 1 metre increase in Wgold, Wbest increases by 0.86

f. In 1984, the *Wbest* value was 2.07 m for a *Wgold* value of 2.02 m.

Show that when this least squares line is fitted to the scatterplot, the residual value for this point is 0.0328   2 marks
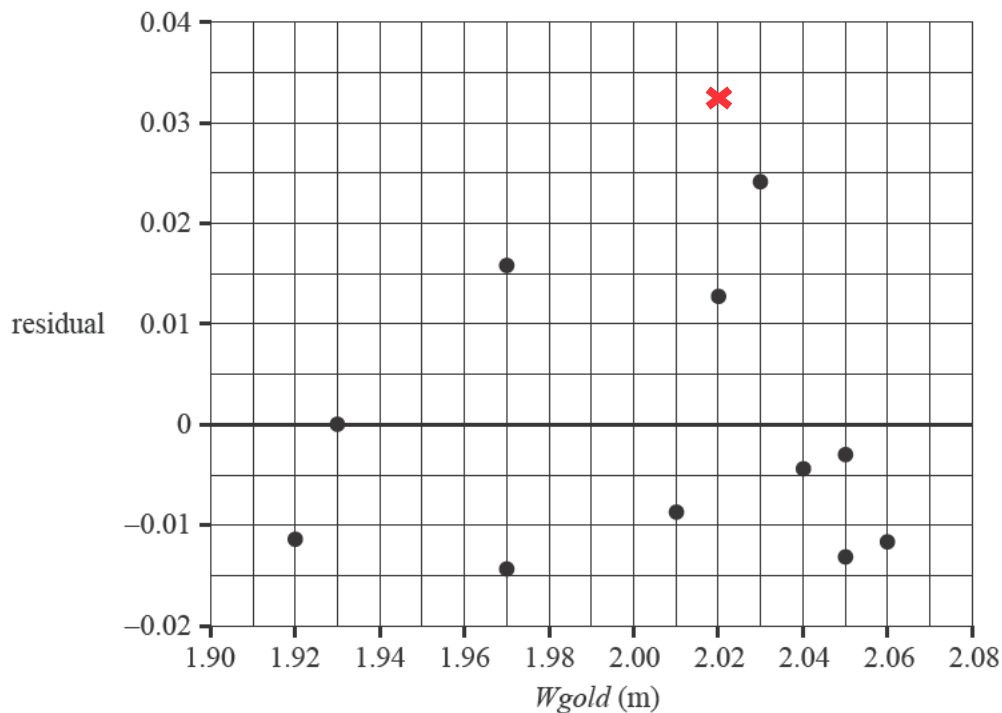
0.3+0.86· x|x=2.02	2.0372
2.07−2.0372	0.0328

predicted value = 0.3+ 0.86 x 2.02 = 2.0372

Residual = 2.07 - 2.0372 = 0.0328

g. The residual plot obtained when the least squares line was fitted to the data is shown below. The residual value from **part f** is missing from the residual plot.

i. Complete the residual plot by adding the residual value from **part f**, drawn as a cross (X), to the residual plot above.   1 mark

ii. In **part b**, a least squares line was fitted to the scatterplot.

Does the residual plot from **part g** justify this? Briefly explain your answer.   1 mark

The residual plot shows no clear pattern therefore it is appropriate to fit a

least squares line.

h. In 1964, the gold medal-winning height, *Wgold*, was 1.90 m. When the least squares line is used to predict *Wbest*, it is found to be 1.934 m.
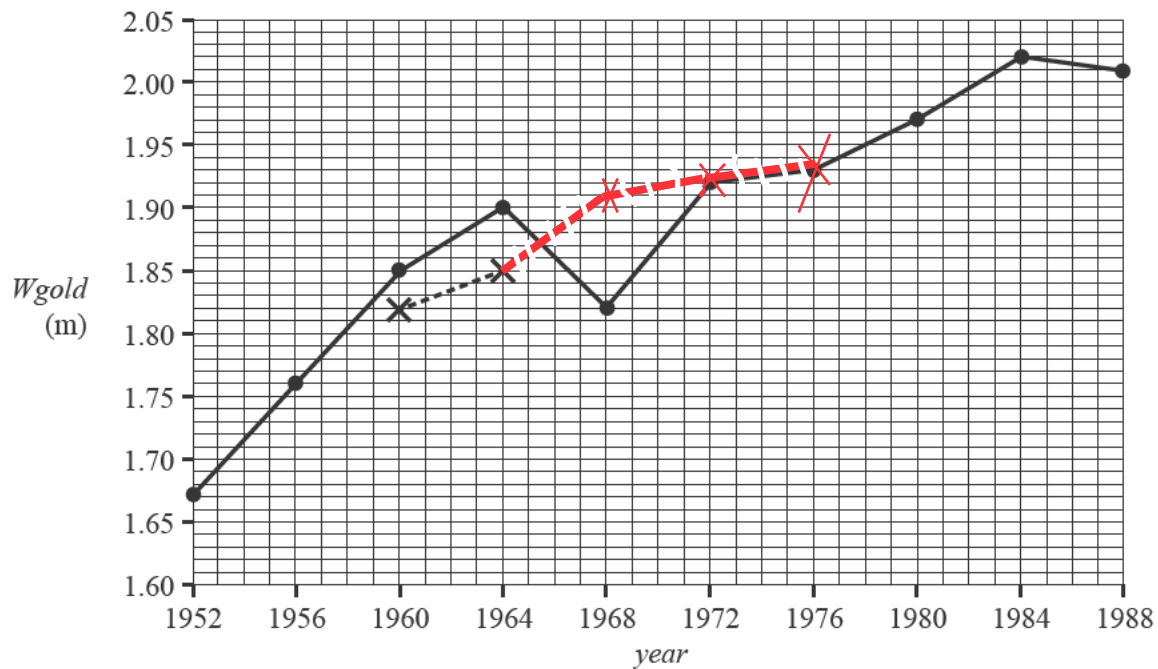
Explain why this prediction is not likely to be reliable.   1 mark

not reliable due to extrapolation

Question 4 (2 marks)

The time series plot below shows the gold medal-winning height for the women's high jump, $Wgold$, in metres, for each Olympic year, $year$, from 1952 to 1988.

Data: <www.olympics.com/en/olympic-games/olympic-results>

A five-median smoothing process will be used to smooth the time series plot above.

The first two points have been placed on the graph with crosses (X) and joined by a dashed line (- - -).

a. Complete the five-median smoothing by marking smoothed values with crosses (X) joined by a dashed line (- - -) on the time series plot above.   1 mark

b. Identify **two** qualitative features that best describe the time series plot above.   1 mark

Increasing trend with regular fluctuations

Recursion and financial modelling

Question 5 (4 marks)

Emi operates a mobile dog-grooming business.

The value of her grooming equipment will depreciate.

Based on average usage, a rule for the value, in dollars, of the equipment, V_n, after n weeks is

$$V_n = 15\,000 - 60n$$

Assume that there are exactly 52 weeks in a year.

a. By what amount, in dollars, does the value of the grooming equipment depreciate each week?   1 mark

$60

b. Emi plans to replace the grooming equipment after four years.

What will be its value, in dollars, at this time?   1 mark

number of weeks in 4 years = 52 x 4 = 208

Value of equipment : 15000 - 60 x 208 = $2520

c. V_n is the value of the grooming equipment, in dollars, after n weeks.

Write a recurrence relation in terms of V_0, V_{n+1} and V_n that can model this value from one week to the next.   1 mark

V₀ = 15 000, Vn+1 = Vn - 60

d. The value of the grooming equipment decreases from one year to the next by the same percentage of the original $15\,000 value.

What is this annual flat rate percentage?   1 mark

weekly interest rate = 60/15000

=0.004

annual flat rate percentage

= 0.04 x 52 x 100%

=20.8%

Question 6 (2 marks)

Emi invested profits of $10 000 into a savings account that earns interest compounding fortnightly, for one year.

The effective interest rate, rounded to two decimal places, is 5.07%.

Assume that there are exactly 26 fortnights in a year.

a. What is the nominal percentage rate of interest for the account?

Round your answer to two decimal places. 1 mark

$\text{solve}\left(\text{eff}(x,26)=5.07,x\right)$ $x=4.95036761696$ *4.95%*

b. Explain why the nominal interest rate appears lower than the effective interest rate. 1 mark

nominal does not take into account the fortnighly compounding

Question 7 (4 marks)

Emi decides to invest a $300 000 inheritance into an annuity.

Let E_n be the balance of Emi's annuity after n months.

A recurrence relation that can model the value of this balance from month to month is

$$E_0 = 300\,000, \qquad E_{n+1} = 1.003E_n - 2159.41$$

a. Showing recursive calculations, determine the balance of the annuity after two months. Round your answer to the ~~nearest cent.~~ 1 mark

$E_0 = 300\ 000$
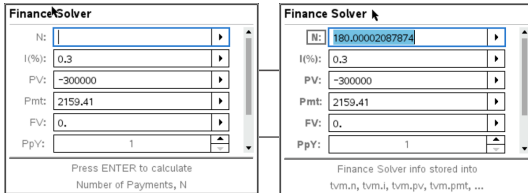$E_1 = 1.003 \times 300\ 000 - 2159.41 = 298\ 740.59$
$E_2 = 1.003 \times 298\ 740.59 - 2159.41 = 297\ 477.401$

$\$297\ 477.40$

300000	300000
Ans· 1.003−2159.41	keep pressing enter
300000	300000
300000· 1.003−2159.41	298740.59
298740.59· 1.003−2159.41	297477.40177

b. For how many years will Emi receive the regular payment? 1 mark

Finance Solver
N: ▯
I(%): 0.3
PV: -300000
Pmt: 2159.41
FV: 0.
PpY: 1
Press ENTER to calculate
Number of Payments, N

Finance Solver
N: 180.00002087874
I(%): 0.3
PV: -300000
Pmt: 2159.41
FV: 0.
PpY: 1
Finance Solver info stored into
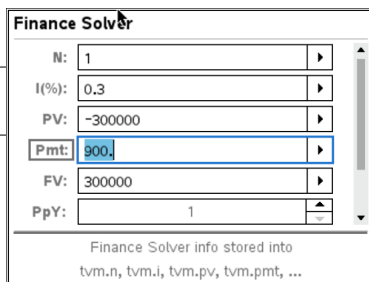tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

180 /12 = 15 years

c. Calculate the annual compound interest rate for this annuity. 1 mark

0.3 x 12 = 3.6%

d. If Emi wanted the annuity to act as a perpetuity, what monthly payment, in dollars, would she receive? 1 mark

Finance Solver
N: 1
I(%): 0.3
PV: -300000
Pmt: 900.
FV: 300000
PpY: 1
Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

$900

Question 8 (2 marks)

Emi takes out a reducing balance loan of $500 000.

The interest rate is 5.3% per annum, compounding monthly.

Emi makes regular monthly repayments of $3071.63 for the duration of the loan, with only the final repayment amount being slightly different from all the other repayments.

Determine the total cost of Emi's loan, rounding your answer to the nearest cent, and state the number of payments required to fully repay the loan.

total cost = | $884

number of payments = | 28

Finance Solver

N:	288.00136298168	▶
I(%):	5.3	▶
PV:	500000	▶
Pmt:	−3071.63	▶
FV:	0	▶
PpY:	12	

Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

pay extra in final payment

Finance Solver

N:	288	▶
I(%):	5.3	▶
PV:	500000	▶
Pmt:	−3071.63	▶
FV:	−4.177344641	▶
PpY:	12	

Finance Solver info stored into
tvm.n, tvm.i, tvm.pv, tvm.pmt, ...

Final Payment
−3071.63 − 4.18
=−3075.81

Total Payments = 287 x 3071.63 + 3075.81

Matrices

Question 9 (3 marks)

Vince works on a construction site.

The amount Vince gets paid depends on the type of shift he works, as shown in the table below.

Shift type	Normal	Overtime	Weekend
Hourly rate of pay ($ per hour)	36	54	72

This information is shown in matrix R below.

$$R = \begin{bmatrix} 36 & 54 & 72 \end{bmatrix}$$

a. Matrix R^T is the transpose of matrix R.

Write down matrix R^T in the space below.   1 mark

$$R^T = \begin{bmatrix} 36 \\ 54 \\ 72 \end{bmatrix}$$

$\begin{bmatrix} 36 & 54 & 72 \end{bmatrix}^T$ $\qquad\qquad$ $\begin{bmatrix} 36 \\ 54 \\ 72 \end{bmatrix}$

During one week, Vince works 28 hours at the normal rate of pay, 6 hours at the overtime rate of pay, and 8 hours at the weekend rate of pay.

b. Complete the following matrix calculation showing the total amount Vince has been paid for this week.   1 mark

$$\begin{bmatrix} \underline{28} & \underline{6} & \underline{8} \end{bmatrix} \times R^T = \begin{bmatrix} \underline{1908} \end{bmatrix}$$

$\begin{bmatrix} 28 & 6 & 8 \end{bmatrix} \cdot \begin{bmatrix} 36 & 54 & 72 \end{bmatrix}^T$ $\qquad\qquad$ $\begin{bmatrix} 1908 \end{bmatrix}$

Vince will receive $90 per hour if he works a public holiday shift.

Matrix Q, as calculated below, can be used to show Vince's hourly rate for each type of shift.

$$Q = n \times \begin{bmatrix} 1 & 1.5 & 2 & p \end{bmatrix}$$
$$= \begin{bmatrix} 36 & 54 & 72 & 90 \end{bmatrix}$$

c. Write the values of n and p in the boxes below. 1 mark

$n =$ | 36

$p =$ | 2.5

solve$\left(n \cdot \begin{bmatrix} 1 & 1.5 & 2 & p \end{bmatrix} = \begin{bmatrix} 36 & 54 & 72 & 90 \end{bmatrix}, n \right)$
$n = 36.$ and $p = 2.5$

Question 10 (2 marks)

To access the southern end of the construction site, Vince must enter a security code consisting of five numbers.

The security code is represented by the row matrix W.

The element in row i and column j of W is w_{ij}.

The elements of W are determined by the rule $(i-j)^2 + 2j$.

a. Complete the following matrix showing the five numbers in the security code.   1 mark

$$W = \left[\begin{array}{ccccc} \underline{\ \ 2\ \ } & \underline{\ \ 5\ \ } & \underline{\ \ 10\ \ } & \underline{\ \ 17\ \ } & \underline{\ \ 26\ \ } \end{array} \right]$$

To access the northern end of the construction site, Vince enters a different security code, consisting of eight numbers.

This security code is represented by the row matrix X.

The element in row i and column j of X is x_{ij}.

The elements of X are also determined by the rule $(i-j)^2 + 2j$.

b. What is the last number in this security code to access the northern end of the construction site?   1 mark

_____65_____

a) $(i-j)^2 + 2 \cdot j | i = 1$ $i^2 + 1$

$i^2 + 1 | j = \{1,2,3,4,5\}$ $\{2,5,10,17,26\}$

b) $i^2 + 1 | j = 8$ 65

Question 11 (3 marks)

A population of a native animal species lives near the construction site.

To ensure that the species is protected, information about the initial female population was collected at the beginning of 2023. The birth rates and the survival rates of the females in this population were also recorded.

This species has a life span of 4 years and the information collected has been categorised into four age groups: 0–1 year, 1–2 years, 2–3 years, and 3–4 years.
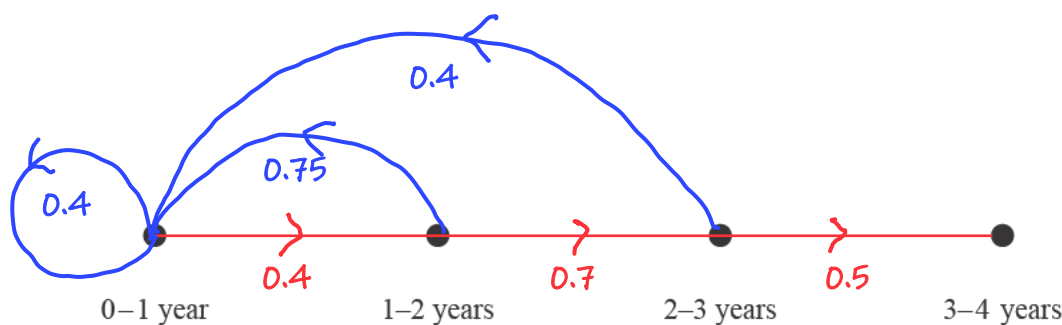
This information is displayed in the initial population matrix, R_0, and the Leslie matrix, L, below.

$$R_0 = \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix} \qquad L = \begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}$$

a. Using the information above

i. complete the following transition diagram.   1 mark

ii. complete the following table, showing the initial female population, and the predicted female population after one year, for each of the age groups.   1 mark

	Age group			
	0–1 year	1–2 years	2–3 years	3–4 years
Initial population	70	80	90	40
Population after one year	124	28	56	45

$$\begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix} \qquad \begin{bmatrix} 124. \\ 28. \\ 56. \\ 45. \end{bmatrix}$$

Question 11 continues on the next page.

b. It is predicted that if this species is not protected, the female population of each of the four age groups will rapidly decrease within the next 10 years.

After how many years is it predicted that the total female population of this species will first be half the initial female population? 　　　　　　　1 mark

5 years

$$[1 \ 1 \ 1 \ 1] \cdot \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix}$$

$[280]$ *initial value*

$$[1 \ 1 \ 1 \ 1] \cdot \begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix} \cdot \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix}$$

$$[253.]$$

$$[1 \ 1 \ 1 \ 1] \cdot \begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}^2 \cdot \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix}$$

$$[190.2]$$

Keep going until sum is less than 140

$$[1 \ 1 \ 1 \ 1] \cdot \begin{bmatrix} 0.4 & 0.75 & 0.4 & 0 \\ 0.4 & 0 & 0 & 0 \\ 0 & 0.7 & 0 & 0 \\ 0 & 0 & 0.5 & 0 \end{bmatrix}^5 \cdot \begin{bmatrix} 70 \\ 80 \\ 90 \\ 40 \end{bmatrix}$$

$$[130.8824]$$

Question 12 (4 marks)

When the construction company established the construction site at the beginning of 2023, it employed 390 staff to work on the site.

The staff comprised 330 construction workers (C), 50 foremen (F) and 10 managers (M).

At the beginning of each year, staff can choose to stay in the same job, move to a different job on the site, or leave the site (L) and not return.

The transition diagram below shows the proportion of staff who are expected to change their job at the site each year.
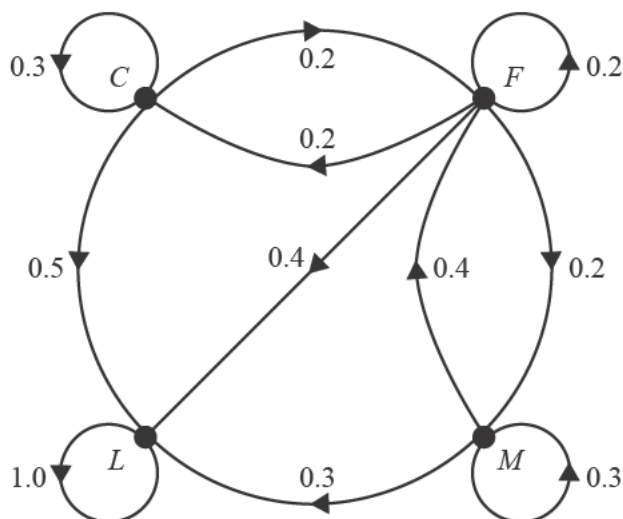
This situation can be modelled by the recurrence relation

$$S_{n+1} = TS_n, \text{ where}$$

T is the transitional matrix, $S_0 = \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} \begin{matrix} C \\ F \\ M \\ L \end{matrix}$ and n is the number of years after 2023.

a. Calculate the predicted percentage decrease in the number of foremen (F) on the site from 2023 to 2025.   1 mark

_____

14%

_____

_____

_____

$\begin{bmatrix} 0.3 & 0.2 & 0 & 0 \\ 0.2 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.5 & 0.4 & 0.3 & 1. \end{bmatrix}^2 \cdot \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 48.7 \\ 43. \\ 19.9 \\ 278.4 \end{bmatrix}$

$\dfrac{43-50}{50} \cdot 100$ −14

Question 12 continues on the next page.

b. Determine the total number of staff on the site in the long term.   1 mark

$$\begin{bmatrix} 0.3 & 0.2 & 0 & 0 \\ 0.2 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.5 & 0.4 & 0.3 & 1. \end{bmatrix}^{50} \cdot \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix}$$

0 (handwritten)

$$\begin{bmatrix} 0.000000000654 \\ 0.000000000981 \\ 0.000000000654 \\ 389.999999998 \end{bmatrix}$$ (handwritten, circled)

To encourage more construction workers (C) to stay, the construction company has given workers an incentive to move into the job of foreman (F).

Matrix R below shows the ways in which staff are expected to change their jobs from year to year with this new incentive in place.

this year

$$R = \begin{array}{cccc} & C & F & M & L \end{array}$$

$$R = \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1 \end{bmatrix} \begin{array}{c} C \\ F \\ M \\ L \end{array} \quad \text{next year}$$

The site always requires at least 330 construction workers.

To ensure that this happens, the company hires an additional 190 construction workers (C) at the beginning of 2024 and each year thereafter.

The matrix V_{n+1} will then be given by

$$V_{n+1} = RV_n + Z, \text{ where}$$

$$V_0 = \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} \begin{array}{c} C \\ F \\ M \\ L \end{array} \qquad Z = \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{array}{c} C \\ F \\ M \\ L \end{array} \quad \text{and } n \text{ is the number of years after 2023.}$$

c. How many more staff are there on the site in 2024 than there were in 2023?   1 mark

need to find out how many leaving (handwritten)

$$\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1. \end{bmatrix} \cdot \begin{bmatrix} 330 \\ 50 \\ 10 \\ 0 \end{bmatrix} + \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 332. \\ 146. \\ 13. \\ 89. \end{bmatrix}$$

190 - 89 = 101 (handwritten)

d.   Based on this new model, the company has realised that in the long term there will be more than 200 foremen on site.

In which year will the number of foremen first be above 200?   1 mark

<u>2027</u>

<u>Use recursion</u>

$$\begin{bmatrix} 33. \\ 50. \\ 10. \\ 0. \end{bmatrix}$$ press Enter

$$\begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.4 & 0.2 & 0.4 & 0 \\ 0 & 0.2 & 0.3 & 0 \\ 0.2 & 0.4 & 0.3 & 1. \end{bmatrix} \cdot \text{Ans} + \begin{bmatrix} 190 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

to post multiply
type matrix R then x ctrl (-)

[×] [ctrl] [(−)]

then + Z enter

$$\begin{bmatrix} 378.2620528 \\ 208.1865608 \\ 54.6561282 \\ 781.8952582 \end{bmatrix}$$
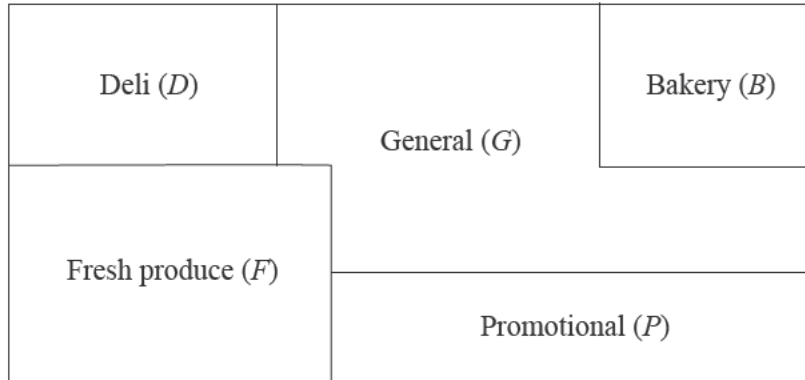
Keep pressing enter until the 2nd row entry is above 200
(don't forget to count the iterations!)

This answer comes after the 5th iteration so over 200 comes during the 4th year i.e 2027

Do not write in this area.

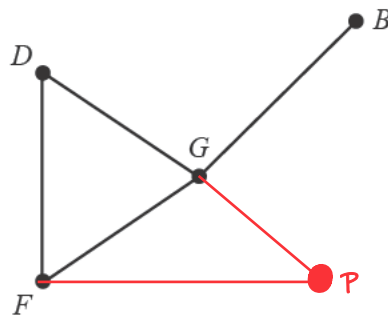## Networks and decision mathematics

Question 13 (4 marks)

A supermarket has five departments, with areas allocated as shown on the floorplan below.

The floorplan is represented by the graph below.

On this graph, vertices represent departments and edges represent boundaries between two departments.

This graph is incomplete.

a. Draw the missing vertex and missing edges on the graph above. Include a label.   1 mark

Karla is standing in the Promotional department.

She wants to visit each department in the supermarket once only.

b.   **i.** In which department will she finish?   1 mark

Bakery

   **ii.** What is the mathematical name for this type of journey?   1 mark

Hamiltonian Path

c. The supermarket adds a new Entertainment department (E), and the floorplan is rearranged.

The boundaries between the departments are represented in the adjacency matrix below, where a '1' indicates a boundary between the departments.

$$
\begin{array}{c}
\begin{array}{cccccc} B & D & E & F & G & P \end{array} \\
\begin{array}{c} B \\ D \\ E \\ F \\ G \\ P \end{array}
\begin{bmatrix}
0 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

Use the adjacency matrix to complete the floorplan below by labelling each department. The Bakery (B) is already labelled.   1 mark
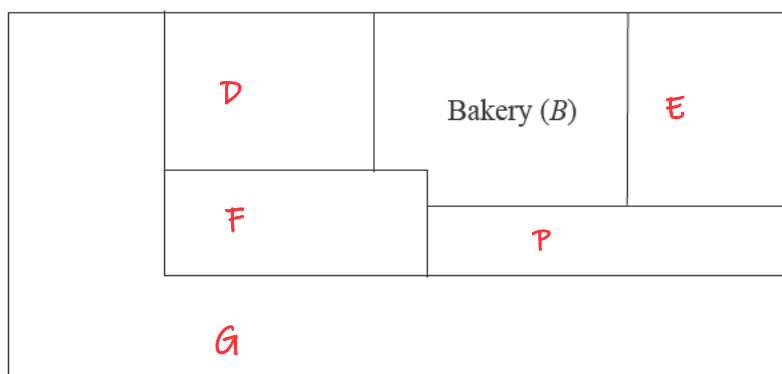
Question 14 (3 marks)

A manufacturer (M) makes deliveries to the supermarket (S) via a number of storage warehouses, L, N, O, P, Q and R. These eight locations are represented as vertices in the network below.

The numbers on the edges represent the maximum number of deliveries that can be made between these locations each day.
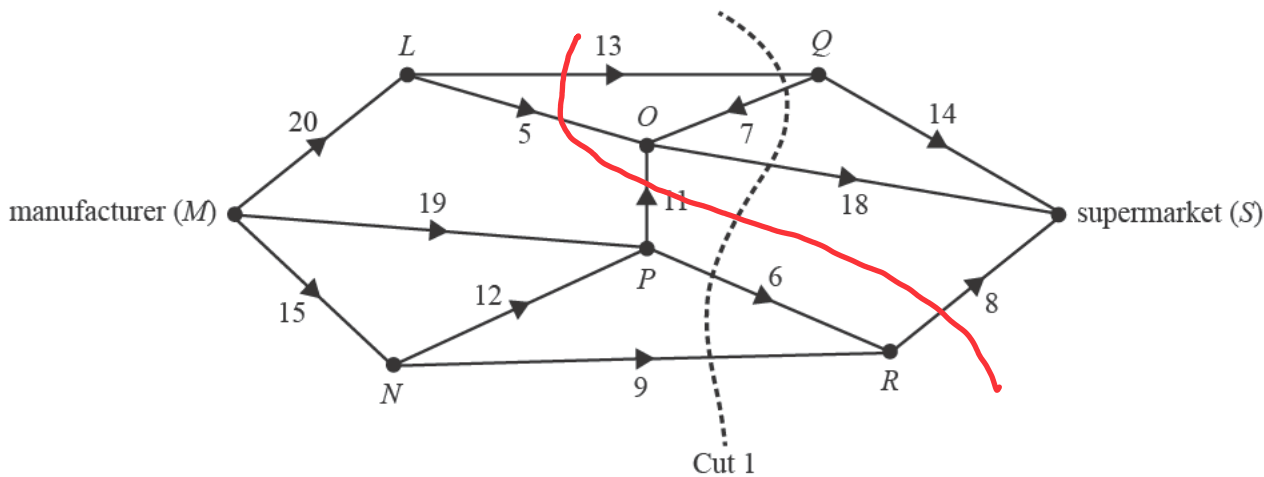
Cut 1

a. When considering the possible flow of deliveries through this network, many different cuts can be made.

Determine the capacity of Cut 1, shown above. 1 mark

$$13 + 18 + 6 + 9 = 46$$

b. Determine the maximum number of deliveries that can be made each day from the manufacturer to the supermarket. 1 mark

minimum cut = maximum flow = $13 + 5 + 11 + 8 = 37$

c. The manufacturer wants to increase the number of deliveries to the supermarket.

This can be achieved by increasing the number of deliveries between one pair of locations.

Complete the following sentence by writing the locations in the boxes provided:

To maximise this increase, the number of deliveries should be increased between
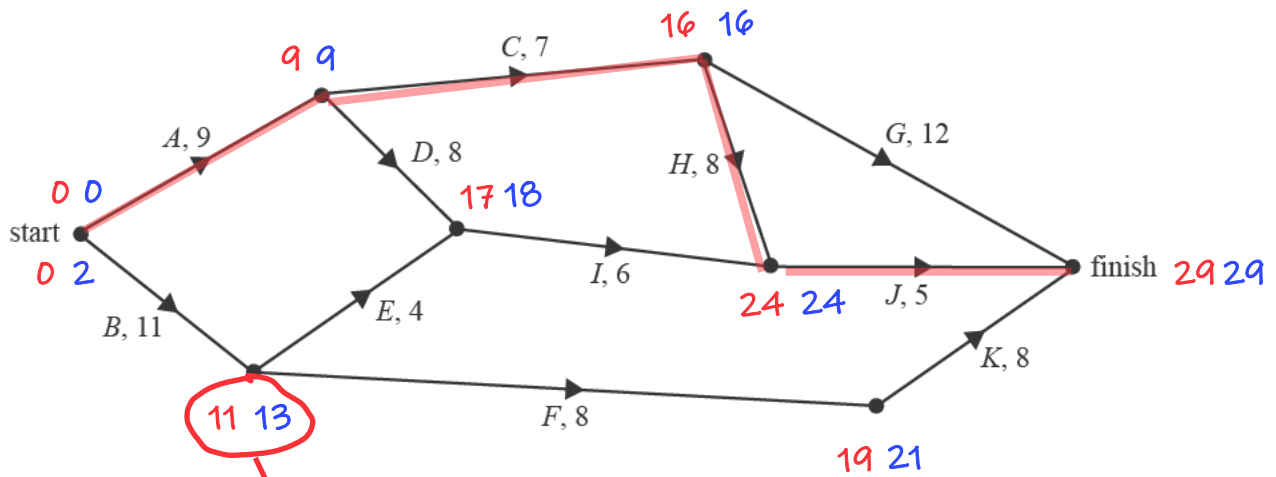
locations ⬚ R and ⬚ S . 1 mark

Question 15 (5 marks)

An upgrade to the supermarket requires the completion of 11 activities, A to K.

The directed network below shows these activities and their completion time, in weeks.

The minimum completion time for the project is 29 weeks.

a. Write down the critical path.   1 mark

A - C - H - J

b. Which activity can be delayed for the longest time without affecting the minimum completion time of the project?   1 mark

E

Use the following information to answer parts c–e.

A change is made to the order of activities.

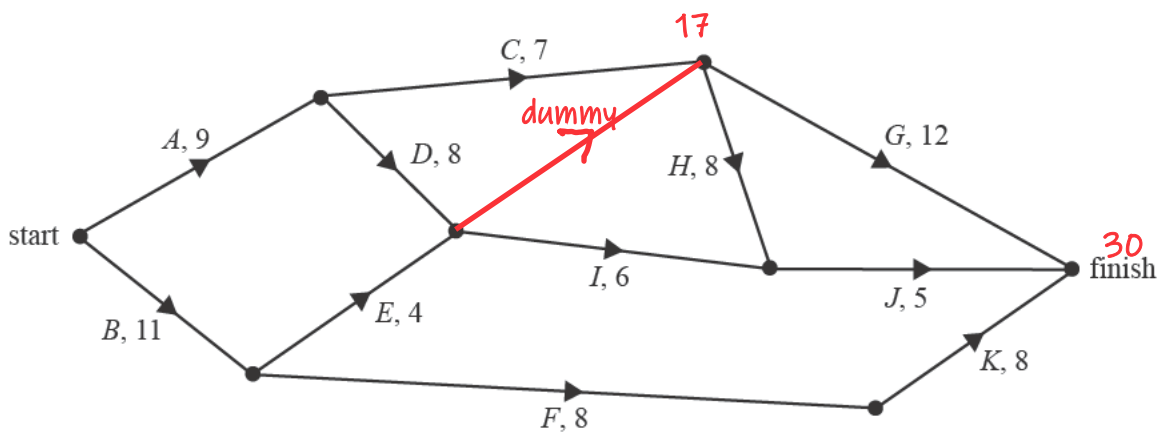The table below shows the activities and their new latest starting times in weeks.

Activity	Latest starting time (weeks)
A	0
B	2
C	10
D	9
E	13
F	14
G	18
H	17
I	19
J	25
K	22

A dummy activity is now required in the network.

c. On the directed network below, draw a directed edge to represent the dummy activity. Include a label.

1 mark

d. What is the new minimum completion time of the project?

1 mark

30 weeks

e.   The owners of the supermarket want the project completed earlier.

They will pay to reduce the time of some of the activities.

A reduction in completion time of an activity will incur an additional cost of $10 000 per week.

Activities can be reduced by a maximum of two weeks.

The minimum number of weeks an activity can be reduced to is seven weeks.

What is the minimum amount the owners of the supermarket will have to pay to reduce the completion time of the project as much as possible?   1 mark

$50 000

Activity	current time	reduced time allowance	cost
A	9	2	10 000 x 2
D	8	1	10 000
H	8	1	10 000
B	11	1	10 000
		Total cost	= 50 000

End of Question and Answer Book

General Mathematics Examination 2

2024 Formula Sheet

You may keep this Formula Sheet.

Data analysis

standardised score	$z = \dfrac{x - \overline{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q1 - 1.5 \times IQR$ upper $Q3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \overline{y} - b\overline{x}$
residual value	residual value = actual value − predicted value
seasonal index	seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a$, $u_{n+1} = Ru_n + d$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n} \right)^n - 1 \right] \times 100\%$

Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \dfrac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \qquad$ where $\qquad \det A \neq 0$
recurrence relation	$S_0 = $ initial state, $\qquad S_{n+1} = T S_n + B$
Leslie matrix recurrence relation	$S_0 = $ initial state, $\qquad S_{n+1} = L S_n$

Networks and decision mathematics

Euler's formula	$v + f = e + 2$

End of Formula Sheet