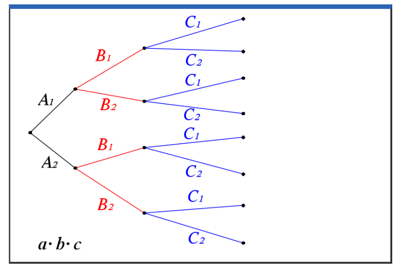


Math Objectives

- Students will explore tree diagrams and expected values.
- Students will be asked to differentiate between independent and dependent events, as well as combining probabilities.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Expected Value
- Tree Diagrams
- Probabilities

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 4 Statistics and Probability:

  4.6: **(a)** Use of Venn Diagrams, tree diagrams, sample space diagrams and tables of outcomes to calculate probabilities.

  (b) Combined events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  (e) Independent events: $P(A \cap B) = P(A) \cdot P(B)$

  4.7: **(b)** Expected value (mean), E(x) for discrete data.

  (c) Applications

  As a result, students will:

  - Apply this information to real world situations.

TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

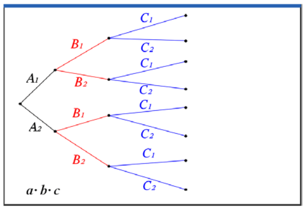
Nspire-ItsToBeExpected-Student.pdf

Nspire-ItsToBeExpected-Student.doc

ItsToBeExpected.tns

ItsToBeExpected_Soln.tns

In this activity, students will use a tree diagram to find theoretical probabilities and use this information in lists to find the expected value. Students will be asked to differentiate between independent and dependent events and how to navigate the handheld when finding these probabilities.



Teacher Tip: You can download the file onto the handhelds or just have the students follow along on the worksheet and your teacher software.

Problem 1 – Creating a Tree Diagram

Open *ItsToBeExpected.*tns. Read the problem on **page 1.2** and follow each page with each question below.

Three basketball players are in a contest, hoping to win money for a charity. There is a 63% chance that Aisha will make a shot, a 74% chance that Bria will make a shot, and a 56% chance that Carmen will make a shot.

1. List the sample space for the three shots. Use an **A**, **B**, or **C** to represent each girl in the sample space.

Solution: (8 outcomes, ^{m} means miss)
ABC, AB^mC, ABC^m, AB^mC^m,A^mBC, A^mB^mC, A^mBC^m, $A^mB^mC^m$

2. Find the probability that **Aisha** will make her shot. Find the probability she will miss her shot.

Solution: 0.63; 0.37

3. Find the probability that **Bria** will make her shot. Find the probability she will miss her shot.

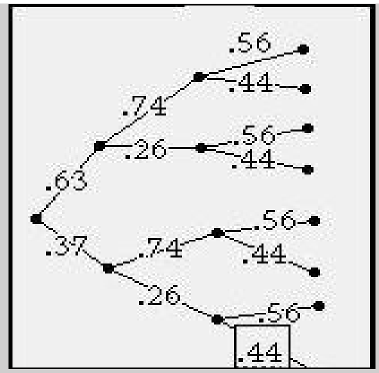
Solution: 0.74; 0.26

4. Find the probability that **Carmen** will make her shot. Find the probability she will miss her shot.

Solution: 0.56; 0.44

One way to organize the results of the scenario is to create a diagram where each girl's shots are represented. Next to the labels of each branch write the appropriate probabilities. (A = Aisha, B = Bria, C = Carmen, 1 = made, 2 = miss.)

Teacher Tip: The diagram given represents one way to calculate the probabilities. Discuss with students other ways to set up the diagram, such as with Bria or Carmen shooting first, and whether that would change the probabilities.

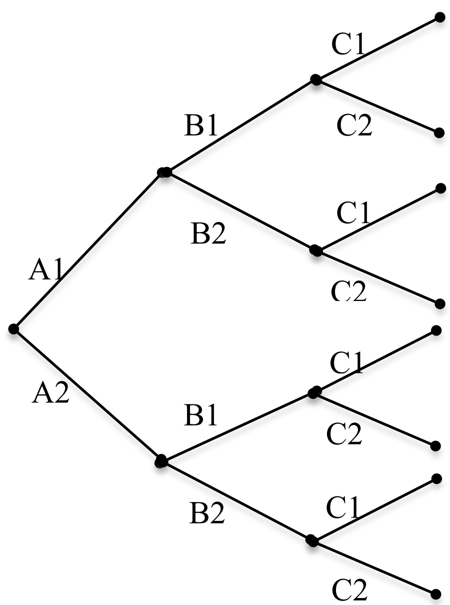


With the help of the diagram, students can calculate the eight probabilities. From these probabilities, they will determine the probabilities of none, one, two, or all three of the girls making their shots. The students should follow a path through the diagram to guide their calculations.

Discuss with students the multiplication rule that allows them to calculate the probabilities. Ask students what other probabilities they can calculate. For example:

Find the probability that Aisha will make her shot and Bria will miss her shot.
((0.63)(0.26) = 0.1638 or 16.38%)

0.63*0.74*0.56	0.261072
0.63*0.74*0.44	0.205128
0.63*0.26*0.56	0.091728
0.63*0.26*0.44	0.072072
0.37*0.74*0.56	0.153328
0.37*0.74*0.44	0.120472
0.37*0.26*0.56	0.053872
0.37*0.26*0.44	0.042328



Since the events of each girl making her shot are independent, the **multiplication rule** for probability can be used. Use the diagram to help calculate the eight probabilities.

5. Find the probability that **none** of the girls make their shots.

Solution: 0.042328

6. Find the probability that **one** girl makes her shot. (*Hint:* Find which of the eight probabilities that must be added together to find the answer.)

Solution: 0.072072+0.12047+0.053872 = 0.246416

7. Find the probability that **two** girls make their shots.

Solution: 0.205128+0.091728+0.153328 = 0.450184

8. Find the probability that **all** the girls make their shots.

Solution: 0.261072

Problem 2 – Introducing Expected Value

Read the problem on **page 1.7**. Then on **page 1.8**, enter the probabilities from **pages 1.5** and **1.6**. Enter the payoff in Column C and then calculate **probabilities · payoff** in Column D.

If only one of the players makes her shot, they earn $5,000. If two make shots, they earn $12,500. If all three are successful, they earn $20,000. All of the money earned goes to charity. You will need to find the expected value of the contest for the charity. **Expected Value** is defined as the sum of the products of probabilities of the outcomes and their payoffs.

9. Find the expected value of the contest.

Solution: $12,080.80

10. State if the charity should expect this amount of money. Explain why or why not.

Possible Discussion: Students may be tempted to say yes based on the term "expected value." Use the tree diagram to show students that it is very possible that the charity will earn *no* money from the contest. The expected value is simply what they would earn on average if the contest were repeated many times.

The following table encompasses the calculations of the entire activity. The first three columns display the chance of each girl making or missing the shot.

Aisha	Bria	Carmen	Probability	Success		
0.63	0.74	0.56	0.261072	3	0.261072	20000
0.63	0.74	0.44	0.261072	2	0.450184	12500
0.63	0.26	0.56	0.091728	2		
0.63	0.26	0.44	0.072072	1	0.246416	5000
0.37	0.74	0.56	0.153328	2		
0.37	0.74	0.44	0.120472	1		
0.37	0.26	0.56	0.053872	1		
0.37	0.26	0.44	0.042328	0	0.042328	0

Teacher Tip: To drive this concept home, calculate the expected value of purchasing a lottery ticket for a 3-digit lottery where the three digits chosen must exactly match the winning digits (order is important). The payoff for matching all three digits in the correct order is $5,000. Otherwise the payoff is $0. There are 720 (${}_{10}P_3$ or P(10,3)) different permutations of the three digits, so the probability of winning is $\frac{1}{720}$ = 0.001389. The expected value is (0.001389 * 5,000) ≈ $7. But should you expect to win $7 each time you buy this type of lottery ticket? No! The probability of winning nothing (and losing the $1 you paid for the ticket) is greater than 99%.

Note: In the lottery described in the Extension problem, the order in which the numbers are chosen does not matter. That problem involves a combination.

Problem 3 – Putting it All Together

In a lottery game, players may pick six numbers from two separate pools of numbers — five different numbers from 1 to 56 and one number from 1 to 46. You win the jackpot by matching all six winning numbers in a drawing.

MATCH		MATCH	PRIZE	CHANCES
5	+	1	Jackpot	1 in 175,711,536
5	+	0	$250,000	1 in 3,904,701
4	+	1	$10,000	1 in 689,065
4	+	0	$150	1 in 15,313
3	+	1	$150	1 in 13,781
3	+	0	$7	1 in 306
2	+	1	$10	1 in 844
1	+	1	$3	1 in 141
0	+	1	$2	1 in 75
Overall chances of winning a prize:				1 in 40

1. Verify the chances to win the jackpot from your knowledge of counting principles.

Solution: See the table below and match the probabilities for each row above.

Prob	Payoff	Prob*Payoff
5.69115E-9	4.2 E7	0.239028
2.56102E-7	250000	0.064025
0.0000001	10000	0.014512
0.000065	150	0.009796
0.000073	150	0.010885
0.003268	7	0.022876
0.001185	10	0.011848
0.007092	3	0.021277
0.013333	2	0.026667

2. Calculate the expected value for the lottery assuming the jackpot is $42 million.

Solution: $0.42

3. Tickets cost $1.00 per play. Find how much the lottery will make/lose for each ticket sold.

Solution: $0.58

4. Find the expected value that would be needed for the lottery to break even.

Solution: $1 (The break-even point is when the expected value is equal to what someone pays to play.)

5. Find what the jackpot would need to be for the lottery to break even.

Solution: Approx. $144,000,000 (To determine the break-even point for mega millions lottery students may use the **solve** command to find the value for ***x*** in the equation):

$(0.181885+\frac{1}{175771536}x-1=0$, or they can solve the equation by hand. Let the jackpot be ***x***.)

Tech Tip: To use the **Numerical Solve** command, press **menu**, **3 Algebra**, **6 Numerical Solve**. Enter the **equation** in the parenthesis**, x**. Press **enter** to solve.

The students can also calculate the number of different sets of five numbers out of 56 numbers by using the **nCr** command, found in the **menu**, **5 Probability**, **3 Combinations**. Enter **(n,r)** in the parenthesis. They should multiply the calculated value by 46 to determine the number of different sets of lottery numbers for this type of lottery.

Note: The calculator uses E to display some of the numbers in scientific notation.

<u>Futher IB Application</u>

After the Bills vs the Patriots game on Sunday, a sample of 50 attendees was randomly selected as they were leaving Highmark Stadium. They were asked how many times they visited the concessions stands for food or drink. The information is summarized in the following frequency table.

Number of times visited concession stands	Frequency
0	5
1	20
2	18
3	4
4	3

It can be assumed that this sample is representative of all attendees to the stadium for next week's game vs the Dolphins. For next week's game, estimate

(a) the probability that a randomly selected attendee will visit a concession stand.

Solution: Summing the frequencies or finding the complement.
Probability = $\frac{45}{50}\ or\ \frac{9}{10}\ or\ 0.90$

(b) the expected number of times an attendee will visit a concession stand.

Solution: Attempt to find the expected value
$\left(0 \times \frac{5}{50}\right) + \left(1 \times \frac{20}{50}\right) + \left(2 \times \frac{18}{50}\right) + \left(3 \times \frac{4}{50}\right) + \left(4 \times \frac{3}{50}\right)$
$\frac{80}{50} = \frac{8}{5} = 1.6\ times$

TI-Nspire Navigator Opportunity: ***Quick Poll (Open Response)*** **Any part to any Problem in the activity would be a great way to quickly assess your student's understanding of finding and discussing both forms of Scientific Notation and Expanded Form.**

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*