About the Lesson

In this activity, students will calculate the experimental probabilities of shooting two free throws, of two people having a shared birthday, and a batter in a baseball getting walked to first base. As a result, students will:

- Understand the difference between experimental and theoretical probability.
- Be able to use simulations to model real events.

Vocabulary

- trial
- experimental probability
- theoretical probability

Teacher Preparation and Notes

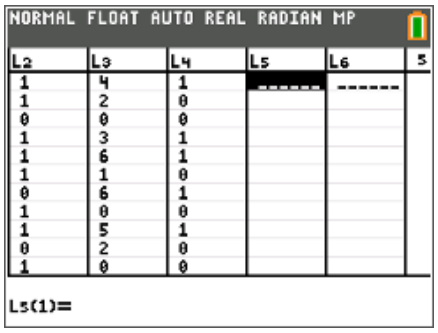
- Because this activity uses random integers, many students will receive different, but correct answers. This is a good opportunity to discuss the idea of randomness.

Activity Materials

- Compatible TI Technologies:

TI-84 Plus*
TI-84 Plus Silver Edition*
TI-84 Plus C Silver Edition
TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- In_It_To_Win_It_Student.pdf
- In_It_To_Win_It_Student.doc

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Problem 1 – Free Throws

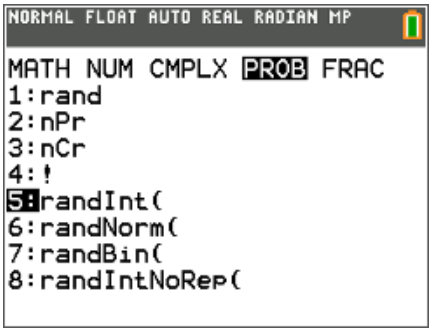
In this problem of the activity, students will simulate a 70% free throw shooter that needs to make both free throws to win the game.

Using lists and formulas, students will find an experimental probability that the shooter will make both free throws to win the game. To open the List Editor, press [STAT] [ENTER].

Students will be using the **randInt** command and the **>** test command. The **randInt** command returns a random integer and will be used to simulate the first and second shots.

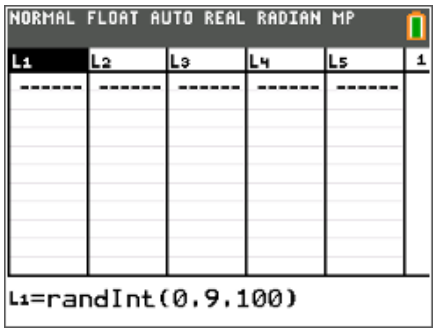
randInt(lowBound, upBound, #Trials)

Lists **L1** and **L3**: **randInt(0,9,100)**



The **randInt** command is found in the **MATH > PRB** menu. This will generate 100 numbers, each representing a free throw. Each row of the list will represent a trial or an attempt to win the game.

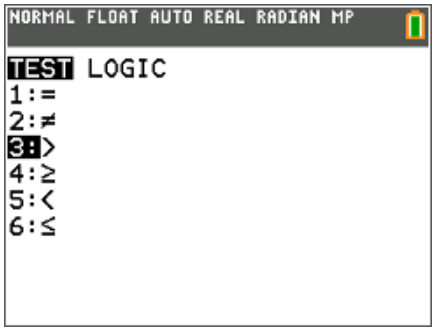
Discuss with students what other numbers could represent a missed basket and a completed basket. They should have three numbers for a miss and seven numbers for a completed basket.

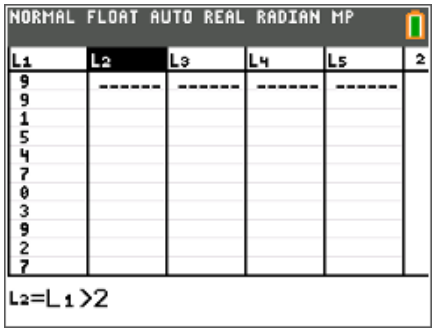


The **>** function will evaluate the values in **L1** and return a Boolean value. The Boolean value will be one if the expression is true and zero if it is false. It will be used to evaluate if the shot is made and then evaluate if a trial is a win for the team. Using the command:

L1>2 will return a 1 for every value in the **L1** that is greater than 2 and a 0 for every value less than 2. (Students may observe that some choices of numbers to represent a miss and a completed basket are better than others. It would be difficult to write an expression that decided automatically if a number in **L1** was a 2, 4, or 6, for example.)

List **L2**: **L1>2** List **L4**: **L3>2**



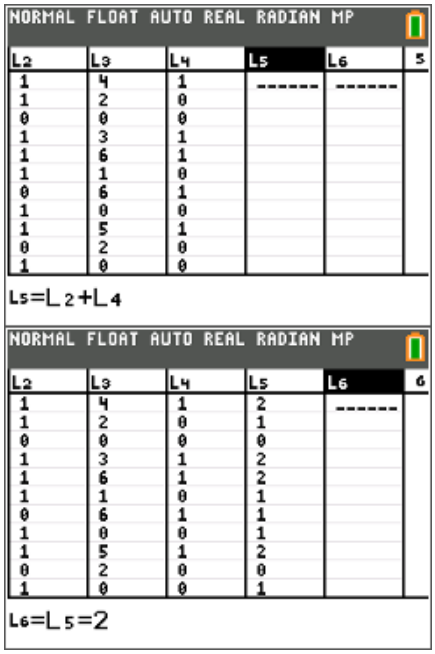


Have students verify that the calculator output matches whether the free throw should be a miss or a completed basket.

Students will look at row 1 and determine how many baskets were completed. Ask students if the team would win if row 1 happened? What number in list **L5** would constitute a win? (2)

Now, students need to assess this for the 100 trials. They will calculate the number of baskets made during each attempt. To do so, they add the numbers in **L2** and **L4**. The added values should be stored in **L5**.

List **L5**: **L2 + L4**

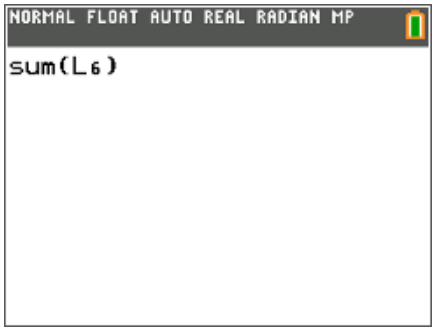


Students will use another Boolean expression to determine if the trial is a win for the basketball team.

List **L6**: **L5 = 2**

The final step of the simulation is to determine the total number of wins for the 100 trials. Since a win is equal to 1, summing the column will give the total number of wins. Students should return to the main screen and enter the command **sum(L6)** to calculate the sum.

Students will divide that number by 100 to get the experimental probability of winning the game.



After completing the simulation individually, encourage students to compare their results first with their group members and then with the entire class.

Note: To repeat the simulation, students will need to re-enter all of the commands for each of the lists **L1** through **L5**. This will overwrite the data in the list.

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To investigate this problem further, have students repeat the experiment with a lower free throw percentage and then again with a higher free throw percentage.

Another investigation could include a free throw percentage where the first shot is 70% and the second shot is 50%.

1. What is the experimental probability that your team won?

Sample Answer: 49%

2. How does it compare to the theoretical probability?

Sample Answer: My experimental probability was close to the theoretical probability of 49%.

3. If the percentage of the free throw shooter decreased to 60%, how much would the probability of your team winning decrease?

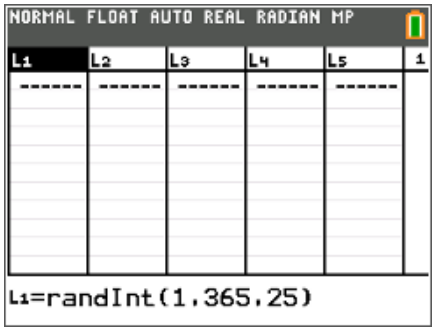
Answer: 36%

4. If the percentage of the free throw shooter increased to 80%, how much would the probability of your team winning increase?

Answer: It would increase by 15% (64%–49%).

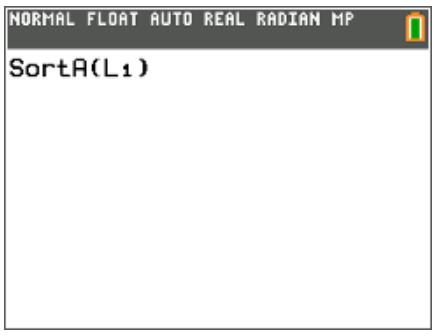
Problem 2 – Birthday Problem

After the discussion, students are to assign an integer to each birthday, assuming that there are 365 days in a year. Because there are 25 people, students are to enter the command **randInt(1,365,25)** in **L1**.



Each integer represents a different day of the year. Have students use the down arrow to scroll down the list and compare the numbers to see if any are the same. Then, students can sort the list to make this inspection easier.

To sort a list, students should choose [STAT] **> SortA(**, type **L1** and press [ENTER]. This will sort **L1** in ascending order. The student can then return to the List Editor to view the result.



5. What is the experimental probability of having a shared birthday?

Sample Answer: 51%

6. How does it compare to the theoretical probability?

Sample Answer: It is very close to the theoretical probability of 56.87%.

7. What happens to the experimental probability if the number of the people in the room increases? Decreases?

Answer: If the number of the people in the room increases, then the experimental probability increases. If the number of the people in the room decreases, then the experimental probability decreases.

8. What birthday was not included in this simulation?

Answer: Leap Day (February 29)

Ask students if they have two people with the same birthday. They are to repeat this procedure 9 more times and record each trial as yes or no, where 'yes' is for one or more shared birthdays and 'no' is for no shared birthdays. The students will need to re-enter the formula and resort the list each of the 9 times. To calculate the experimental probability, students should divide the total number of yes's by 10. They can then compare their probability with those in their group and the class.

The theoretical probability is approximately 57%.

1 – P(no shared) = P(1 shared)

$$1-\frac{365!}{(365-n)!\cdot 365^n}=1-\frac{365!}{(365-25)!\cdot 365^{25}}=0.5687$$

An extension to this problem could include changing the number of people in the room or including February 29th, making the number of possible birthdays 366.

Extension – Casey at the Bat

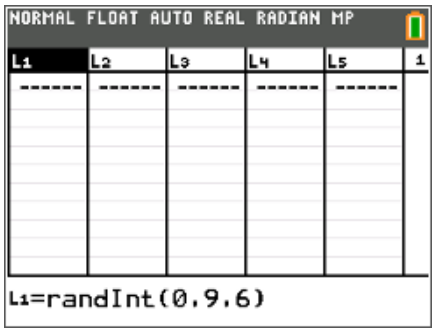
There is a famous poem titled *Casey at the Bat* by Ernest Lawrence Thayer. It can be found at the end of this document. You can choose to have students read it, but it is not required.

Students should create a simulation that represents the scenario. They will execute the simulation 25 times and record for each time yes or no, 'yes' meaning that Casey walked and 'no' meaning that Casey struck out. 'Yes' will be represented by 1 and 'no' signified by a 0. They may record the number of yes's and no's on paper or they may use lists and increment the value when appropriate. The experimental probability is 25 divided by the number of yes's.

9. Discuss ideas with your group. As a group, decide on the specifics for the simulation. Record them below.

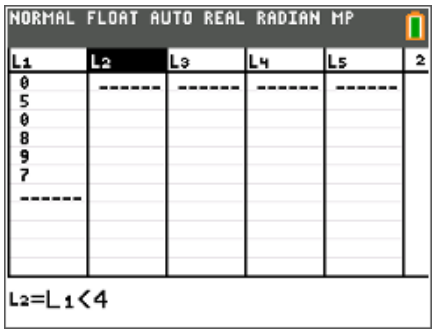
Answer: The pitcher throws strikes 40% and balls 60% of the time. Casey should swing the bat if the pitch is a strike; otherwise, take the walk.

One way to simulate the situation is to use the command **randInt(0,9,6)** A strike is represented by numbers 0-3 and a ball by numbers 4-9. Casey will receive a maximum of six pitches.



Students can then use the formula **L1<4** in **L2** to determine if a pitch is a ball or a strike. A one represents an out and a zero represents a ball. If four 0s occur before three 1s, then Casey walks. If three 1s occur before four 0s, then Casey strikes out.

To recalculate the simulation for the remaining 24 trials, students can highlight **L1** and re-enter the formula.



Further investigation could include determining what percentage of strikes the picture throws in order for Casey to have a 75% chance of walking to first base, etc.

10. Perform your simulation 25 times and record the experimental probability below.

Sample Answer: Casey would have drawn a walk 95% of the time.

11. How does the experimental probability compare to the theoretical probability?

Sample Answer: It was very close.

12. Did the coach make a good decision?

Answer: Yes, there was a greater probability for Casey being walked that being struck out.

Casey at the Bat
By Ernest Lawrence Thayer
Taken From the San Francisco Examiner - June 3, 1888

The outlook wasn't brilliant for the Mudville nine that day;
The score stood four to two, with but one inning more to play,
And then when Cooney died at first, and Barrows did the same,
A pall-like silence fell upon the patrons of the game.

A straggling few got up to go in deep despair. The rest
Clung to that hope which springs eternal in the human breast;
They thought, "If only Casey could but get a whack at that —
We'd put up even money now, with Casey at the bat."

But Flynn preceded Casey, as did also Jimmy Blake,
And the former was a hoodoo, while the latter was a cake;
So upon that stricken multitude grim melancholy sat;
For there seemed but little chance of Casey getting to the bat.

But Flynn let drive a single, to the wonderment of all,
And Blake, the much despised, tore the cover off the ball;
And when the dust had lifted, and men saw what had occurred,
There was Jimmy safe at second and Flynn a-hugging third.

Then from five thousand throats and more there rose a lusty yell;
It rumbled through the valley, it rattled in the dell;
It pounded on the mountain and recoiled upon the flat,
For Casey, mighty Casey, was advancing to the bat.

There was ease in Casey's manner as he stepped into his place;
There was pride in Casey's bearing and a smile lit Casey's face.
And when, responding to the cheers, he lightly doffed his hat,
No stranger in the crowd could doubt 'twas Casey at the bat.

Ten thousand eyes were on him as he rubbed his hands with dirt.
Five thousand tongues applauded when he wiped them on his shirt.
Then while the writhing pitcher ground the ball into his hip,
Defiance flashed in Casey's eye, a sneer curled Casey's lip.

And now the leather-covered sphere came hurtling through the air,
And Casey stood a-watching it in haughty grandeur there.
Close by the sturdy batsman the ball unheeded sped —
"That ain't my style," said Casey. "Strike one!" the umpire said.

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From the benches, black with people, there went up a muffled roar,
Like the beating of the storm-waves on a stern and distant shore;
"Kill him! Kill the umpire!" shouted some one on the stand;
And it's likely they'd have killed him had not Casey raised his hand.

With a smile of Christian charity great Casey's visage shone;
He stilled the rising tumult; he bade the game go on;
He signaled to the pitcher, and once more the dun sphere flew;
But Casey still ignored it, and the umpire said "Strike two!"

"Fraud!" cried the maddened thousands, and echo answered "Fraud!"
But one scornful look from Casey and the audience was awed.
They saw his face grow stern and cold, they saw his muscles strain,
And they knew that Casey wouldn't let that ball go by again.

The sneer has fled from Casey's lip, the teeth are clenched in hate;
He pounds with cruel violence his bat upon the plate.
And now the pitcher holds the ball, and now he lets it go,
And now the air is shattered by the force of Casey's blow.

Oh, somewhere in this favored land the sun is shining bright,
The band is playing somewhere, and somewhere hearts are light,
And somewhere men are laughing, and little children shout;
But there is no joy in Mudville — mighty Casey has struck out.