

Means with Confidence

ID: 12370

 Time required
 15 minutes

Activity Overview

In this activity, students will estimate the true mean of a population when the standard deviation is known by finding the sample mean, margin of error and confidence interval. They will also use the **ZInterval** command to confirm the intervals.

Topic: Inference

- *Standardized normal curves*
- *Confidence Intervals*
- *Estimating a population mean*

Teacher Preparation and Notes

- *Problem 1 on the worksheet is a review about the normal distribution. Depending on the level of students Problem 1 can be skipped and the activity can begin with Problem 2.*
- *Students should know how to calculate the mean of a sample.*
- ***To download the student worksheet and data lists AVE, LONG, and YAC, go to education.ti.com/exchange and enter "12370" in the keyword search box.***

Associated Materials

- *MeansWithConfidence_Student.doc*
- *AVE, LONG, YAC (lists)*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Estimating a Population Proportion (TI-84 Plus family) — 9985*
- *Are You Confident? (TI-Nspire technology) — 9676*
- *Hypothesis Testing: Means (TI-84 Plus family) — 10239*
- *Estimating and Finding Confidence Intervals (TI-84 Plus family) — 4588*

Problem 1 – Review of Normal Distribution and z-scores

Review the normal curve and the percent of data that falls within one, two, and three standard deviations of the mean. There are questions included in the first part of the activity to check students' knowledge of this topic. To answer the questions students should assume a normal distribution with a mean of 50 and standard deviation of 5.

Introduce the Central Limit Theorem: The Central Limit Theorem says that given a SRS (simple random sample) of size n from any population with mean μ and finite standard deviation σ , when n is large, the sampling distribution of the sample mean \bar{x} is close to the normal distribution with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$.

Discuss how this theorem allows one to find an interval that contains the mean of the population.

Discussion Questions:

- *If we have the mean of the sample, why would we want the mean of the population?*
- *What information do we need to use the Central Limit Theorem to approximate the mean?*
- *If we would like to be certain 95% of the time, how many standard deviations from the mean can we be?*

Worksheet Answers:

1. c. 68%

2. c. 47.5%

3. c. (40, 60)

4. a. 5th

Questions 5 and 6 allow students to interact with the following information.

Introduce the two parameters needed for a confidence interval:

- A confidence level which gives the probability that the interval will contain μ .
- An interval calculated from the data, generally written $\bar{x} \pm \text{Error}$.

Error = $Z \cdot \frac{\sigma}{\sqrt{n}}$, where Z is the z-score that corresponds to the area in the left (or right) tail.

Worksheet Answers:

5. b. (2.5, 97.5)

6. c. 90th

Problem 2 – Estimating the true mean

This problem can be used as either an introduction or review for estimating the population mean. If using as an introduction, you will need to guide students through each step as more specific calculator instructions are not given on the worksheet.

The data in the lists **AVE**, **LONG**, and **YAC** is a sample of 40 receivers from the population of all receivers in a hypothetical football league. Included in the sample is the average reception (**AVE**), the longest reception (**LONG**), and the yards after completion (**YAC**).

This data is repeated in the table on the next page. One row is one receiver's stats.

Ave	Long	Yac	Ave	Long	Yac
2.9	8	6.5	10	14	2.5
11.5	54	12.3	8.3	17	4.6
10.8	28	5.5	12.3	47	4.5
7.2	15	4.2	10.7	36	3.6
15.5	49	6	16.7	70	4.3
8.3	11	12.7	7.8	25	4.5
13.7	65	4.2	14.8	20	9.8
13.1	24	4.1	3.3	10	4.3
11.6	58	3.4	1.5	2	0
9.2	25	5.8	9.2	79	10.9
11.5	30	3.8	19	19	21
17.9	51	2.2	10.2	26	1.6
-7	-7	0	10	15	10.3
7.7	14	3.4	11	31	6
9.5	29	2.3	3.7	12	3
12.3	20	12	-2	-2	3
10	44	6.8	9	9	7
9	19	2.4	14.2	60	2.8
6.8	18	5.1	9.4	29	3.6
11.2	40	3.3	13.8	48	2.7

Students will use the **LONG** sample data to estimate the mean for the longest reception for all receivers with a confidence interval of 95%. The standard deviation is known to be 20.39.

Step 1: Calculate \bar{x} for longest reception.

This can be done two ways on the *Calculator* application on page 2.4.

1. Enter **sum(LLONG)/40**. This adds all the values in Column C and divides by n (shown at right). To do this, press **[2nd]** **[LIST]**, arrow to the MATH menu and select the command. Then press **[2nd]** **[LIST]** again to select the list **LONG**.
2. Calculate the One-Variable Statistics and find \bar{x} . Press **[STAT]**, arrow to the CALC menu and select the command. Then enter the list.

Students should store (**[STO▶]**) the value they calculate as **X** for easy calculations later.

```
sum(LLONG)/40
      29.05
Ans→X
      29.05
```

```
1-Var Stats LON
G
```

Step 2: Find the critical value.

Students can use the **invNorm** command (**[2nd]** **[DISTR]**). This command will return the z-score for a given percentile when given μ and σ , in this case $\mu = 0$ and $\sigma = 1$.

Since students are finding a 95% confidence interval, the percentile (area) is either 2.5% or 97.5%.

Step 3: Calculate the error.

Students are given the standard deviation for all receivers as 20.39. They should use this to calculate the error.

$$(z = 1.96, \sigma = 20.39, \text{ and } n = 40) \quad \text{Error} = Z \cdot \frac{\sigma}{\sqrt{n}}$$

Students should store the value they calculate for the error as **E** for easy calculations later.

```
0: normalcdf(
1: normalcdf(
2: invNorm(
3: invT(
4: tpdf(
5: tcdf(
6: x²pdf(
```

```
invNorm(.975,0,1
)
      1.959963986
Ans*(20.39/√(40)
)
      6.318810359
Ans→E
```

If using Mathprint OS:

Students can calculate the error by displaying it as a stacked fraction. To do this, press **[ALPHA]** **[F1]** and select **n/d**. Then enter the value of the numerator, press **[↓]** to move to the bottom of the fraction, enter the value of the denominator and press **[ENTER]**.

```
invNorm(.975,0,
)
      1.959963986
Ans* $\frac{20.39}{\sqrt{40}}$ 
      6.318810359
```

Step 4: Write the solution as an interval.

Students can use the earlier determined values to calculate the confidence interval.

Step 5: Interpret the result as it pertains to the problem situation.

Sample answer: I am 95% confident that the mean of the receivers' longest reception is between 22.73 and 35.37.

Alternative method:

After completing the process, show students the built-in calculator function, **ZInterval**, to calculate the confidence interval using the given statistics.

Students can choose **[STAT]**, arrow to the TESTS menu and select **ZInterval**. For the Inpt, students need to choose **Stats**. Then they need to enter σ , \bar{x} , n , and the confidence level as shown.

Then press **[ENTER]** on Calculate.

```
X-E      22.73118964
X+E      35.36881036
```

```
ZInterval
Inpt:Data Stats
σ:20.39
x̄:29.05
n:40
C-Level:.95
Calculate
```

```
ZInterval
(22.731, 35.369)
x̄=29.05
n=40
```

Homework Problems

Students are given homework problems that ask them to find different confidence intervals using the sample data from lists **AVE**, **LONG**, and **YAC**. Then can either find the interval using the method outlined in Problem 2 or using the **ZInterval** command.

1. 1.26
2. (9.62, 11.06)
3. (9.49, 11.20)
4. (25.72, 33.12)
5. (4.85, 5.90)

You can have students compare their estimations with the true means and have them determine if the confidence interval will always or sometimes contain the true mean when the standard deviation of the population mean is known.

```
invNorm(.9,0,1)
1.281551567
Ans*(6.24/√(40))
1.264418029
```

The population means are:
 Average reception: 10.344
 Longest reception: 29.417
 Yards after completion: 5.372