Math Objectives

For positive values of *x,* students will identify the following behaviors of exponential and power functions:

- For large $(x > a)$ *x*-values, exponential functions of the form $y = a^x$ grow faster than power functions of the form $y = x^a$.
- For particular *x*-values, power and exponential functions can be equivalent.
- On certain intervals, power functions can have greater value than exponential functions.

Vocabulary

- exponential function
- power function
- exponent
- base

About the Lesson

- This lesson involves comparing rates of growth between the exponential function $f(x) = a^x$ and the power function $g(x) = x^a$ for positive *x*-values.
- As a result, students will:
  - Compare the discrete value of these functions for *a* = 2, 3, 4, and 5 as *x* moves along a number line from 1 to 5.
  - Compare the graphs of the functions for *a* = 2, 3, 4, and 5 as *x* moves along the *x*-axis.

TI-Nspire™ Navigator™ System

- Use Quick Poll to assess students' understanding throughout the activity.
- Use Live Presenter for student demonstrations.
- Use Screen Capture to examine patterns that emerge.
- Use Teacher Software to review student documents.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Use a minimized slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Exponential_vs_Power_Student.pdf

Exponential_vs_Power_Student.doc

TI-Nspire document

Exponential_vs_Power.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

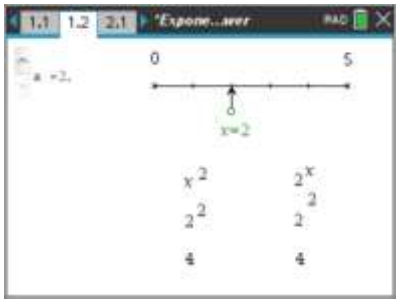
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☜) getting ready to grab the point. Then press ctrl ☜ to grab the point and close the hand (☜).

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Teacher Tip: Since there is some intentional mathematical ambiguity in question 1, consider working through it as a class.

Move to page 1.2.



1. Compare the functions $f(x) = a^x$ and $g(x) = x^a$ when $a = 2$ by dragging point x along the number line.
   a. As x increases, which function appears to grow faster?

   Sample answer: It is difficult to tell for these values alone. There are times when the power function is greater, when the exponential function is greater, and even when they are equal.

   b. For what x-values, if any, are the functions 2^x and x^2 equal?

   Answer: They are equal for $x = 2$ and $x = 4$.

2. Explore several different a-values using Δ and ∇. As you do so, continue to drag point x along the number line.
   a. As x increases, does the exponential function or the power function appear to grow faster?

   Answer: Regardless of the base, students should see that sometimes the power function is greater, and sometimes the exponential function is greater.

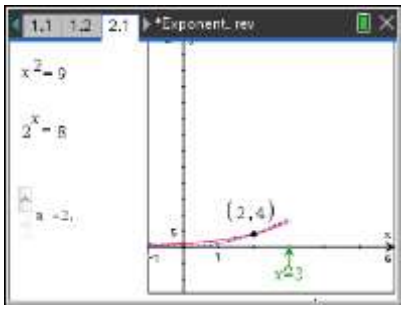
b. For what *x*-values, if any, are the functions equal? Summarize your results in the table below.

Answer:

Base	*x*-values
2	2, 4
3	3
4	2, 4
5	5

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 2 at the end of this lesson.

Move to page 2.1.



3. Drag the point *x* on the arrow to the right to produce two graphs, one solid and one dashed.

a. Identify which graph represents the exponential function $f(x) = 2^x$ and which graph represents the power function $g(x) = x^2$. Justify your answer.

Answer: The solid graph represents $f(x) = 2^x$ and the dashed is $g(x) = x^2$. Students should notice that $x^2 > 2^x$ between $x = 2$ and $x = 4$. Likewise, when $x > 4$, $x^2 < 2^x$.

b. As *x* increases, does the exponential function or the power function appear to grow faster?

Sample answer: As *x* grows larger, sometimes the power function is greater and sometimes the exponential function is greater.

c. For what *x*-values greater than 0, if any, are the functions equal?

Answer: The functions are equal at $x = 2$ and $x = 4$.

d. Are there any other x-values for which the two functions are equal?

Sample answer: Students should notice that there will not be another positive value where the functions intersect, but some might notice that the functions intersect for a negative *x*-value.

TI-Nspire Navigator Opportunity: *Live Presenter, Quick Poll (Open Response),* and *Screen Capture*
See Note 3 at the end of this lesson.

Teacher Tip: Students should share their conjectures as a class and decide if there is a negative *x*-value for which the two functions are equal.

Teacher Tip: For larger base values, the negative *x*-values may be more difficult to see. This can elicit discussion as to what occurs when *x* is negative in each function.

4. Explore several different *a*-values using ∆ and ∇. As you do so, continue to drag point *x* along the number line.

a. Complete the table below for $x > 0$.

Answers:

a	Interval(s) where $a^x < x^a$	Interval(s) where $a^x > x^a$
2	$2 < x < 4$ or (2, 4)	$0 < x < 2$, $x > 4$ or $(0, 2) \cup (4, \infty)$
3	$2.48 < x < 3$ or (2.48, 3)	$0 < x < 2.48$, $x > 3$ or $(0, 2.48) \cup (3, \infty)$
4	$2 < x < 4$ or (2, 4)	$0 < x < 2$, $x > 4$ or $(0, 2) \cup (4, \infty)$
5	$1.76 < x < 5$ or (1.76, 5)	$0 < x < 1.76$, $x > 5$ or $(0,1.76) \cup (5, \infty)$

Teacher Tip: For values of $a > 2$, it may be difficult for students to determine the answers. Students may have to move the x value slowly in order to watch the values on the left.

b. In general, for large values of *x,* which increases faster: an exponential function or a power function?

Answer: An exponential function increases faster for larger *x*-values.

Teacher Tip: Students should share their conjectures as a class and discuss, as a class, that for even *a*-values there will be another intersection point, but for odd *a*-values there are none due to the shapes of the graphs.

Teacher Tip: You may want to discuss with your students that in the case of exponential functions of the form a^x, large *x*-values need only be greater than *a.* That is, once $x > a$ you have $a^x > x^a$. For example, 4 is a large number for $f(x) = 2^x$ but not for $g(x) = 5^x$.

TI-Nspire Navigator Opportunity: *Quick Polls (Open Response)* and *Screen Capture*
See Note 4 at the end of this lesson.

5. You plan to invest money for *x* number of years. You get to choose whether your interest is calculated using the function $f(x) = 4^x$ or $g(x) = x^4$. Which would you choose and why?

Sample answers: Students should answer that if the money will be invested between 0 and 2 years then $f(x) = 4^x$ would be better. If the money is invested for exactly 2 years, both functions are equally good. If the money will be invested between 2 and 4 years, then $g(x) = x^4$ would be better. If the money is invested for exactly 4 years, both functions are equally good. If the money will be invested for more than 4 years, then $f(x) = 4^x$ is best.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- Exponential functions grow faster than power functions for large *x*-values.
- Power and exponential functions can be equal for particular *x*-values.
- Power functions can actually be greater than exponential functions on some intervals.

Assessment

Question 5 could be used as an assessment item.

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Note 1

Start of Lesson, *Live Presenter:* *Live Presenter* can be used to have a student or the teacher demonstrate how to drag and move the point *x* along the number line.

Note 2

Question 2b *Screen Capture*: Take a *Screen Capture* of page 1.2 where students are on different *a*-values. As a class, discuss the various cases that occur.

Note 3

Question 3 *Live Presenter*, *Quick Poll (Open Response)*, and *Screen Capture*: *Live Presenter* can be used to show students how to grab and drag the point *x* as well as to identify which graph is the exponential function and which graph is the power function.

Send an *Open Response Quick Poll*, asking students to submit their answers to questions 3c and 3d.

If students have difficulty identifying where the functions are equal, take a *Screen Capture* of page 2.1. As a class, discuss the *x*-coordinates of the points of intersection indicate when the functions are equal.

Note 4

Question 5 *Quick Polls* (*Open Response*) and *Screen Capture:* Send an *Open Response Quick Poll*, asking students to submit their answer to question 5.

Take a *Screen Capture* of page 2.1. As a class, have students justify their answer.