First Principles

Teacher Notes & Answers

7 8 9 10 **11** 12

TI-30XPlus MathPrint™ | Worksheet | Student | 45 min

Gradient at a Point

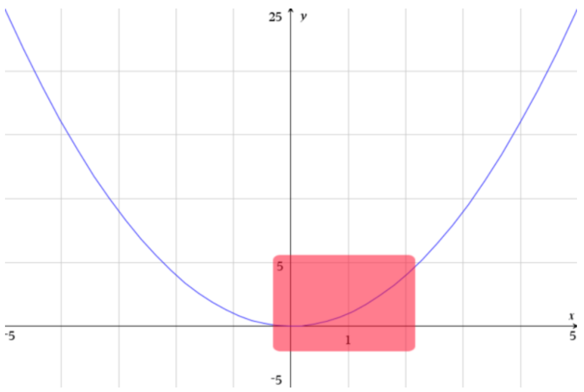
Teacher Notes:

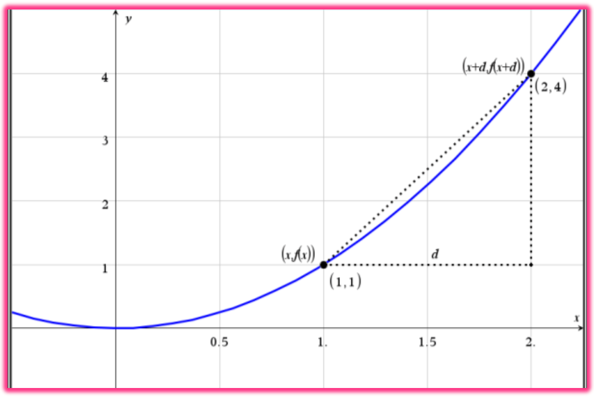
Dynamic visual representations can help students understand the concept of differentiation from first principles and represent an ideal compliment to this activity. The images used in this activity have been generated using TI-Nspire™ computer software.

The function notation on the TI-30XPlus-MathPrint is a useful tool that will help students chunk their thinking and thereby reducing cognitive overload.

A graph of $y = x^2$ is shown below. The graph on the right represents the highlighted region. We are interested in the gradient of the curve at the point (1, 1). The gradient of the secant from (1,1) to (2, 4) can be used to estimate the gradient at the point (1, 1) and allows for a traditional approach for calculating the gradient.

Gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$ or using function notation: $\dfrac{f(x+d) - f(x)}{(x+d) - x} = \dfrac{f(x+d) - f(x)}{d}$





Question: 1.

i) Calculate the gradient of the line joining the points (1, 1) and (2, 4).
Answer: Gradient = 3.

ii) Will your answer to part (i) be more or less than the gradient at the point (1, 1)?
Answer: The secant is steeper, so it's gradient will be more than the gradient at the point (1, 1).

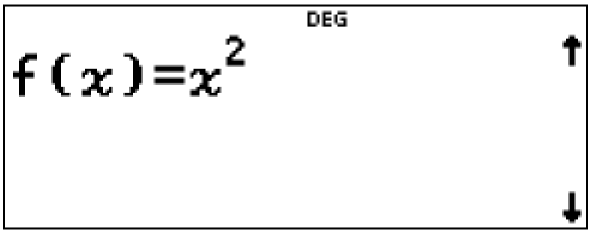
iii) The point (2, 4) is moved to (1.5, 2.25). Will the gradient of the secant provide a better (or worse) approximation for the gradient at the point (1, 1)?
Answer: The approximation will be better, closer to the gradient at (1, 1).

iv) Determine the gradient of the secant joining the points (1, 1) and (1.5, 2.25).
Answer: Gradient = 2.5

Author: P. Fox

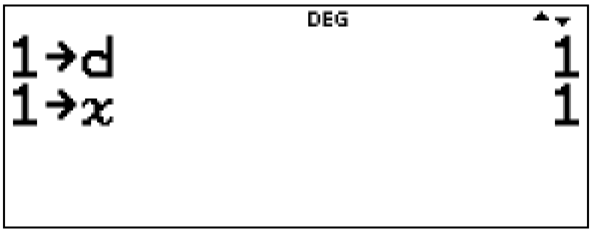
Using your calculator – Gradient at a point

Define the function $f(x)=x^2$ on your calculator.



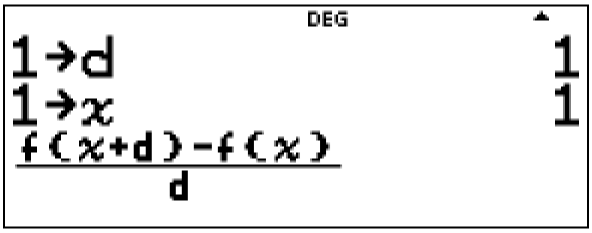
Exit the function editor and return to the main screen.

To begin, store ‘1’ in memory location: d.

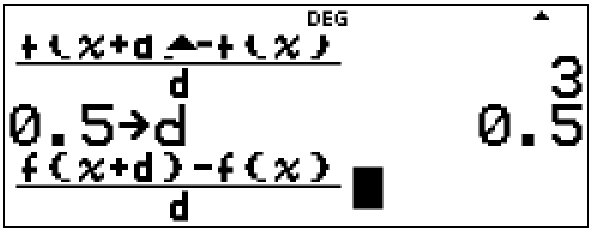


And also store 1 in x.

Enter the expression for the gradient using the fraction key and use the corresponding function.
Check your answer to Question 1(i)



Now store 0.5 in d, then copy and pasted the gradient equation and recalculate.
Compare your result to Question 1(iv).



Question: 2.

Use your calculator to explore the gradient of the secant as *d* gets smaller and smaller and the two points get closer and closer together.

d	1	0.5	0.1	0.01	0.001
Gradient of Secant	3	2.5	2.1	2.01	2.001

Question: 3.

As *d* gets smaller, what is the gradient of the line passing through (1, 1) approaching?

Answer: Gradient is approaching 2.

Question: 4.

Repeat the calculations from Question 2 to explore the gradient on the curve of the point (2, 4).
Change the stored value for x to 2.

d	1	0.5	0.1	0.01	0.001
Gradient of Secant	5	4.5	4.1	4.01	4.001

Answer: Gradient is approaching 4.

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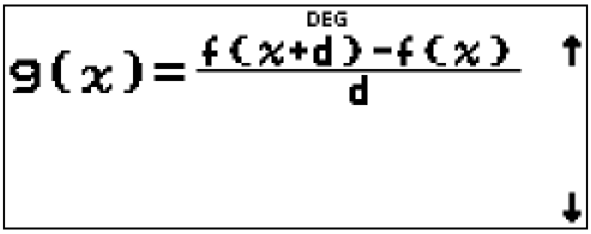
Defining a Function for the Gradient

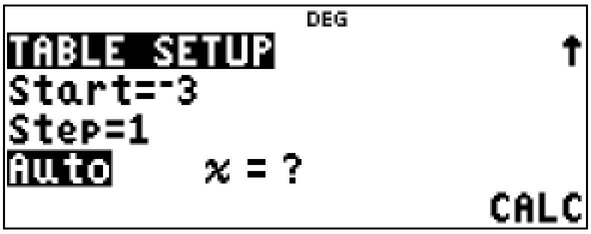
With d set to 0.001, the gradient of the secant provides a good estimate for the gradient of a point on the curve. Use table key to return to the function editor. Navigate to g(x) and define it the same as the gradient from the calculator's home screen.



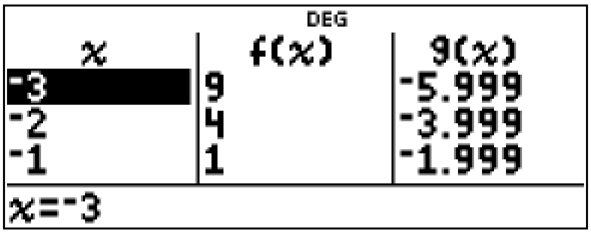
Once the gradient function is defined press: enter

A table of values can be generated automatically for both f(x) and gradient which is now defined in g(x).

Set the table to start at -3 in steps of 1.



A sample section of the table is shown opposite. Check to make sure your results are the same. (d = 0.001)



Question: 5.

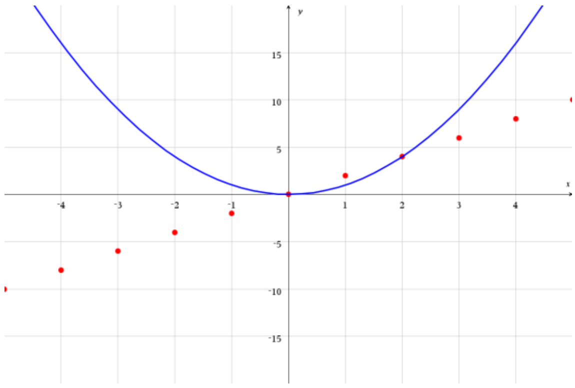
Scroll through the table of values and study the relationship between x and the gradient of the function $f(x)=x^2$.

i) Graph the function $f(x)=x^2$ on the axis provided. **Answer**: See Graph Opposite

ii) Plot the points corresponding to the gradient of the function. **Answer**: See Graph Opposite

iii) Write a rule for the gradient function in the form: $f'(x)=$ where $f'(x)$ represents the gradient of the function $f(x)$.

Answer: $f'(x)=2x$



Question: 6.

Change the defined function on the calculator to: $f(x)=x^2+1$. Explore the table of values for the gradient function and compare it to the gradient function for $f(x)=x^2$. Suggest reasons for the result.

Answer: The gradient is the same for all values of x, the shape of the graph is exactly the same, the transformation, parallel to the y axis doesn't change the gradient.

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Question: 7.

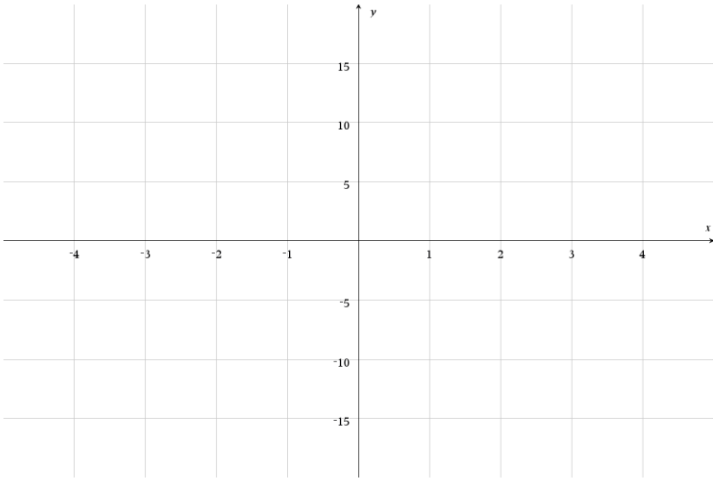
Change the defined function on the calculator to: $f(x)=(x-1)^2$. Explore the table of values for the gradient function and compare it to the gradient function for $f(x)=x^2$. Suggest reasons for the result.

Answer: The gradient function is $f'(x)=2(x-1)$, the idea here is for students to focus on the fact that the original function has been translated so the gradient function has simply followed it across, similar to the previous question whereby students focus on the translation and how that would affect the gradient.

Question: 8.

Change the defined function on the calculator to: $f(x)=x^3$. Explore the table of values for the gradient function, graph the results and determine a function for the gradient.

Answer: Students should identify the gradient function as $f(x)=3x^2$



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