

Student Activity

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Introduction

CNEOS = Centre for Near Earth Object Studies, “Near Earth Objects” include comets and asteroids that have been attracted by the Earth’s gravitational field. These objects are like time capsules; their composition remains relatively unchanged since the formation of the universe. The proximity and accessibility of NEOs makes them cheaper to access than planetary missions. The other significant interest in NEOs is the issue of whether or not they are likely to pose any impact risk with Earth. There are two programs that monitor potential threats:

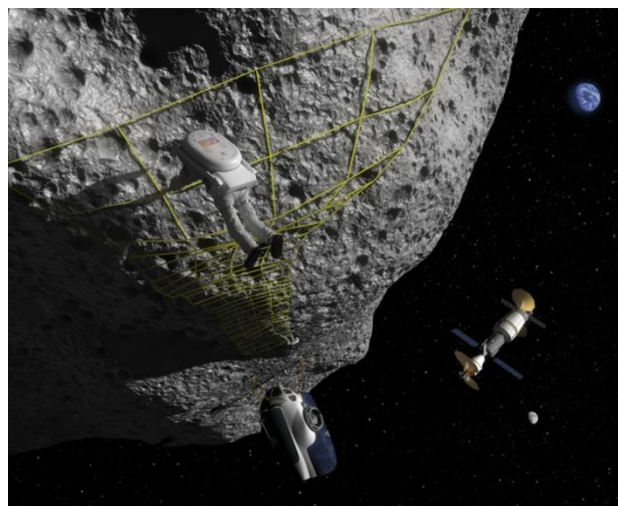


Image Courtesy of NASA/JPL-Caltech

Sentry Designed by CNEOS, this system monitors known NEOs for both short term and long term impact risk.

Scout This system monitors the short term impact risk for unconfirmed NEOs.

In this investigation you will explore a highly simplified mathematical model of these monitoring systems to determine:

- Shortest distance between a curve and a point
- Shortest distance between two curves

The shortest distance between two points on a flat surface is a straight line; space is three dimensional and curved! In this investigation we consider only the path of the object, in space these objects are moving, so too the reference frame from which the objects are observed. Despite the enormous differences between the complex Sentry system and the simplified problem presented here, the concepts developed in this investigation form a basis for understanding the combination of calculus and geometric concepts.

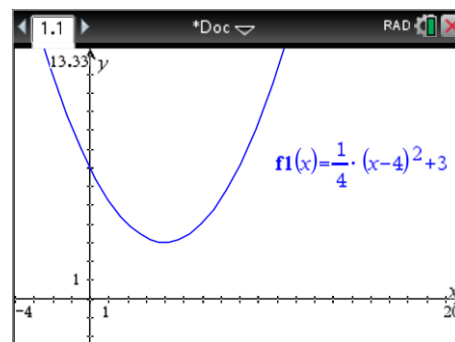
Shortest Distance between a Point and a Curve

A great way to solve a challenging problem is to start with an easier one. Create a new TI-Nspire document and insert a Graph Application. Graph the function:

$$f(x) = \frac{1}{4}(x-4)^2 + 3$$

Adjust the Window settings:

Menu: Window / Zoom > Zoom – Quadrant 1



Question: 1.

What is the distance between the origin (0, 0) and the y axis intercept?

Question: 2.

What is the distance between the origin (0, 0) and the turning point on the curve?

A line segment can be used to measure the straight line distance between the origin and a point on the curve.

Menu: Geometry > Points & Lines > Segment

When the line segment tool is active click on the origin (point of intersection) followed by a point on the curve (point on), as shown opposite.

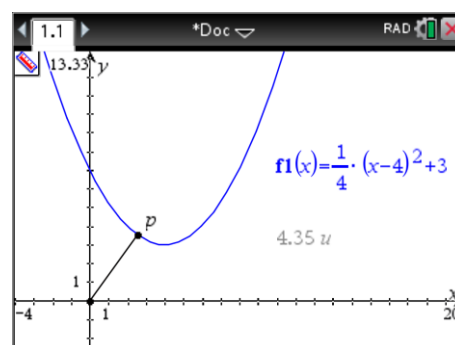
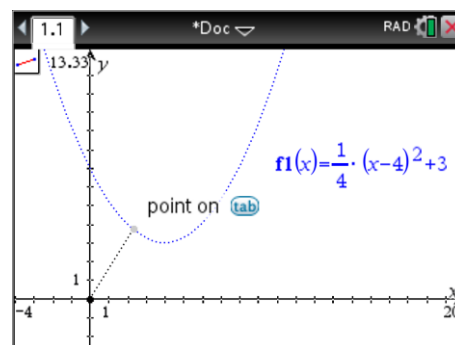
Note: This will be referred to as point P.

The length of the line can be computed automatically.

Menu: Geometry > Measurement > Length

Click on the line segment then move the mouse to where you would like the measurement displayed and click again.

Press [ESC] to release the measurement tool. Place the mouse over point P on the graph, grab the point and move it along the graph. Observe the change in length and corresponding measurement.

**Question: 3.**

Experiment with point P to find the shortest distance. According to your experimentation, what is the shortest distance between the origin and the point P on the curve?

Question: 4.

The coordinates of point P corresponding to the shortest experimental distance can be obtained easily on the calculator. Place the mouse over point P and press [Ctrl] + [Menu], select Coordinates and Equations.

- What are the coordinates of your point?
- What is the gradient of this line segment?

Question: 5.

Calculate the gradient of the curve at the experimental point P.

Question: 6.

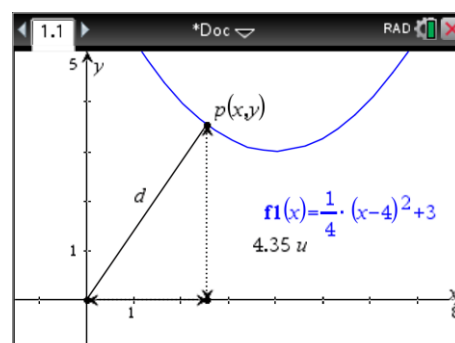
Calculate the product of the gradient of the curve at point P (Q.5) and the gradient of the line segment (Q.4b). Remembering that these values are approximate, what does your answer suggest?

The diagram opposite shows an enlarged and labelled version of the graph and line segment. The length of the line segment can be expressed as a function: $d(x)$ by using Pythagoras' theorem.

The base of the triangle has length x

The height of the triangle has length y where $y = \frac{1}{4}(x-4)^2 + 3$

Define a function $d(x)$ using this information.



Question: 7.

Use calculus to determine the minimum distance between the origin and point P on the curve.

Question: 8.

Determine the coordinates of point P corresponding to the minimum distance and calculate the gradient of the corresponding line segment.

Question: 9.

Use calculus to determine the gradient of $f(x)$ at point P, corresponding to the minimum distance.

Question: 10.

Calculate the product of the gradient (Q.9) and the line segment (Q.8). What does this result suggest about the relationship between the curve and the line segment?

Shortest Distance between two Curves

Objects in space generally travel with a curved trajectory as they are influenced by the gravitational attraction of other bodies. Concepts developed in the previous questions will help explore this somewhat more challenging problem of finding the shortest distance between two curves. Open the TI-Nspire file: CNEOS.

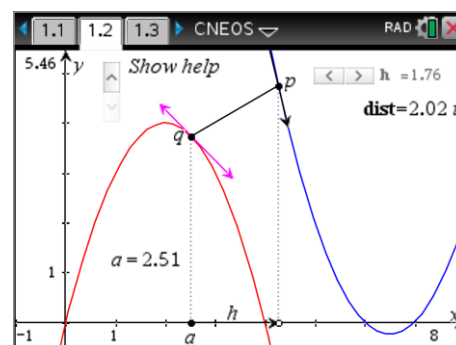
Navigate to page 1.2.

Point a can be moved along the x -axis and corresponds to point q on the function $f(x) = 4x - x^2$.

Point p is on the function $g(x) = x^2 - 13x + 42$ and is h units horizontally from q . Point p can be moved by adjusting the slider that controls h .

The length of the line segment \overline{pq} is displayed as 'dist'.

A help option is available; when selected this will display the result of a calculation. The calculation method is not disclosed, but the value displayed may be helpful.

**Question: 11.**

Drag point a so that it has an x -coordinate of approximately 2.5. Use the slider to adjust the value of h until the distance between points p and q is a minimum. (Experimentally)

- What is the distance between the two points?
- Explain how this problem relates to the problem explored previously.

Question: 12.

Drag point a so that it has an x -coordinate of approximately 3. Use the slider to adjust the value of h until the distance between points p and q is a minimum. (Experimentally)

- What is the distance between the two points?
- Based on your results from Q11a, Q12b and the visual representation of the problem, estimate the value of a for which the distance between the two curves will be a minimum.
- If points p and q are located such that the distance between $g(x)$ and $f(x)$ is a minimum, describe the relationship between the tangents to $g(x)$ and $f(x)$ at p and q .

Question: 13.

Determine an expression for the gradient of $g(x)$ at any point x .

Question: 14.

Determine an expression for the gradient of $f(x)$ at any point $x + h$.

Question: 15.

Using your conclusions from Q12.c, determine an expression for h in terms of x .

Question: 16.

Use Pythagoras's theorem and the diagram opposite to determine a function $d(x)$ in terms of x .

Question: 17.

Determine the minimum value of the distance function and the corresponding coordinates of points p and q .

