Concepts

These documents are designed to introduce some of the key concepts associated with the rectilinear, or straight line, motion of an object. These ideas are examined in the context of the vertical motion of an elevator. The connection to the vertical coordinate of related graphs is more natural in this context because it does not require additional visual conversion associated with horizontal motion.

The Elevator Height as Integral of Velocity documents allow the user to manipulate a piecewise linear function and to define an arbitrary function that represents the velocity of the elevator. In the first file, Elevator_Height_PT1.tns, the initial height, or starting point, of the elevator is $h = 0$ at time $t = 0$, where h is measured in floors and t in seconds. The height, or position, of the elevator is dynamic, changed by using the slider in the top left, associated with t.

In the second file, Elevator_Height_PT2.tns, the user can enter an arbitrary function for velocity. The position of the elevator is still controlled by the slider for t. Both files also show graphs of the position function on a separate page, and the graph of the velocity and position function simultaneously on another page.

Course and Exam Description Unit

Section 4.2: Straight-Line Motion: Connecting Position, Velocity, and Acceleration

Calculator Files

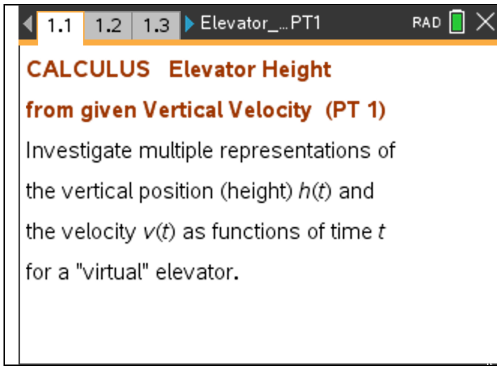
- Elevator_Height_PT1.tns
- Elevator_Height_PT2.tns

Using the Documents

The two calculator files differ in how the user sets, or defines, the velocity function for the elevator.

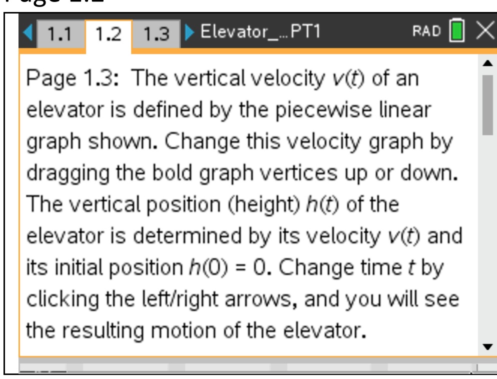
Elevator_Height_PT1.tns: The velocity of the elevator is presented as a continuous piecewise linear graph. The vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing any marked point on the graph and dragging to another location. The vertical motion of the elevator is illustrated in the left pane as the value of t is changed, either by using the *clicker* (minimized slider) or by grabbing and dragging the point on the horizontal axis corresponding to t.

Page 1.1



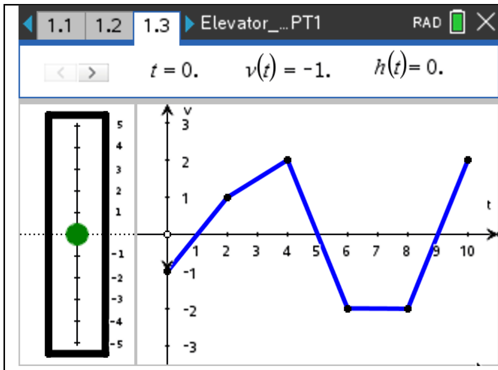
This introductory page explains the purpose of this tns file: to investigate the relationship between the velocity function and the vertical position, or height, of an elevator.

Page 1.2



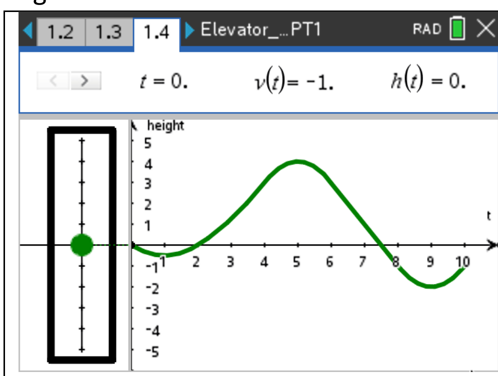
This page describes the remaining calculator pages in more detail. There are also function definitions and Mathboxes at the bottom of this page that are used in background calculations on other pages. These are essential to the tns file, so they should not be edited or changed. Interested teachers and students might consider their purpose and how they contribute to the output.

Page 1.3



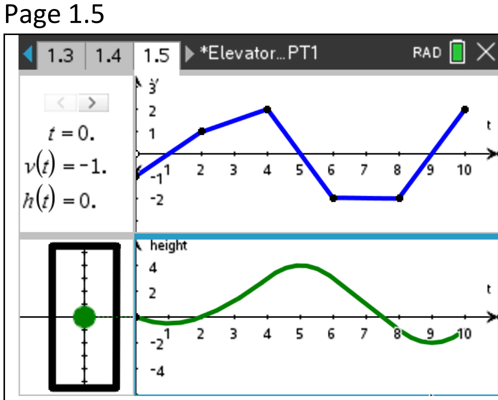
The graph of the velocity of the elevator for $0 \le t \le 10$ seconds is displayed in the bottom right pane. The vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing the point and dragging to another location. Values of t, $v(t)$, and $h(t)$ (in floors) are given in the top pane. The motion of the elevator is illustrated (in the left pane) as the value of t is changed, either by using the *clicker* (minimized slider) or by grabbing and dragging the point on the horizontal axis corresponding to t. At each value of t, there is an arrow drawn to the corresponding point on the velocity curve which represents the direction the elevator. The initial floor for the elevator is 0.

Page 1.4



The graph of the position of the elevator (floor) for $0 \le t \le 10$ (seconds) is displayed in the bottom right pane. It is also simultaneously illustrated in the left pane. Values of t, $v(t)$, and $h(t)$ are given in the top pane. The motion of the elevator is illustrated as the value of t is changed, either by using the clicker or by grabbing and dragging the point on the horizontal axis corresponding to t. The vertical axis in the bottom right pane for height corresponds to the height of the elevator in the left pane, as indicated by the dotted line.

Page 1.5



The graph of the velocity and the position of the elevator, for $0 \le t \le 10$ (seconds), are displayed simultaneously in the right two panes. The motion of the elevator is illustrated in the bottom left pane and the values of t, $v(t)$, and $h(t)$ are given in the top left pane. Also on this page, the motion of the elevator can be illustrated as the value of t is changed, here only by using the clicker. The vertices that connect the linear pieces of the velocity graph can be moved up or down (in integer steps) by grabbing the point and dragging to another location.

Suggested Applications and Extensions

Elevator_Height_PT1.tns
Use the default velocity function in considering the following questions. Remember that t is measured in seconds, and h in floors. On page 1.3, observe the numerical values of v and h in the top pane, and the motion of the elevator in the left pane, as t varies from 0 to 10. On page 1.4, the graph of the height of the elevator is displayed and the motion of the elevator is depicted in the left pane. Page 1.5 includes both graphs so that the velocity and height function can be viewed simultaneously. Note that the elevator can descend to basement floors.

1. Use the velocity function to explain when the elevator is moving up. Moving down.
2. Use the velocity function to explain when the elevator changes direction.
3. Find the time at which the elevator is at its highest point. Lowest point. What is the value of the velocity at those times?
4. When is the elevator moving fastest? Slowest?
5. Find the average velocity of the elevator over the time interval $0 \le t \le 10$.
6. Estimate the total distance traveled by the elevator over the time interval $0 \le t \le 10$.
7. Estimate the displacement of the elevator over the interval $0 \le t \le 10$. Explain why this value and the total distance traveled are different. Could these two values be the same? If so, how?
8. Use the values for $h(t)$ on page 1.4 to estimate the slope of the tangent line to the graph of h at $t = 2$. Compare this value with $v(2)$.
9. Find the value of $h(5) - h(1)$. Explain the meaning of this value in the context of this problem. Use geometry to find the area of the region bounded by the graph of v and the t-axis between $t = 1$ and $t = 5$. How do these two values compare?
10. Find the value of $h(9) - h(5)$. Explain the meaning of this value in the context of this problem. Use geometry to find the area of the region bounded by the graph of v and the t-axis between $t = 5$ and $t = 9$. How do these two values compare?
11. Use the display on page 1.5 to explain how the graphs of v and h are related.

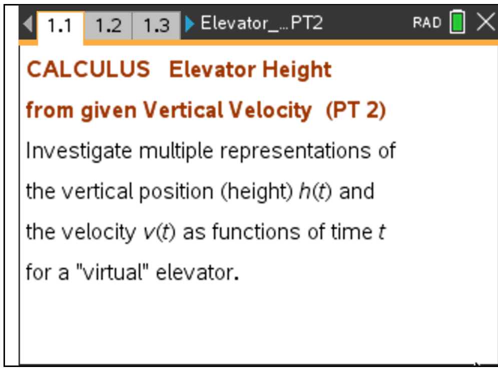
Remember that h is measured in floors and t is measured in seconds. On page 1.3, the vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing the point and dragging to another location.

1. Construct a velocity graph so that the elevator moves to the fourth floor in 4 seconds, remains there for 2 seconds, then moves down to $h = 0$ in 4 seconds.
2. Construct a velocity graph such that the elevator ends on the fourth floor in the basement after 10 seconds.
3. Construct a velocity graph such that the elevator changes direction twice and travels a total of 14 floors in 10 seconds.
4. Is it possible to construct a velocity graph such that the elevator is always moving up but never moves higher than floor 6? Why or why not?

5. Construct a velocity graph such that the elevator is at $h=0$ after 10 seconds. Consider the area of the region(s) bounded by the graph of v and the t-axis between $t=0$ and $t=10$. Explain how these areas can be used to compute the height of the elevator at $t=10$ seconds. Test your conjecture using other velocity curves and different values for $h(10)$. Describe a general method for using the velocity curve to find the height of the elevator after 10 seconds.

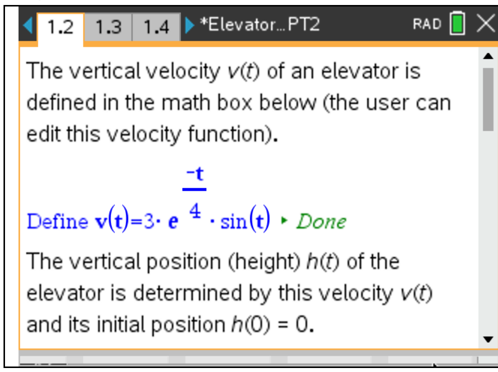
Elevator_Height_PT2.tns:

Page 1.1



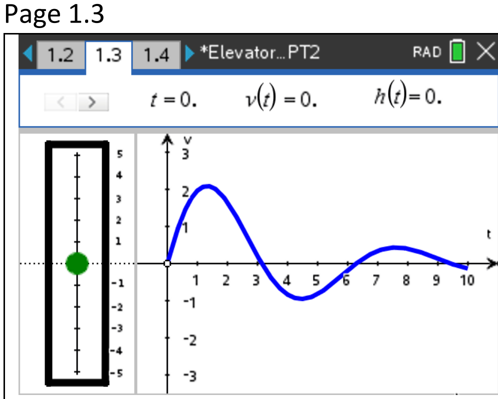
This introductory page explains the purpose of this tns file: to investigate the relationship between the velocity function and the vertical position, or height, of an elevator. In this file, the velocity is not restricted to a piecewise linear function, and the user can define an arbitrary velocity function analytically.

Page 1.2



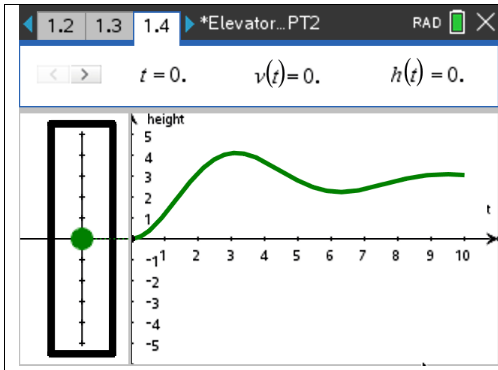
The velocity of the elevator is defined in a Mathbox, and the default function is shown in the figure. The user can change the expression for v at any time, on this page. The remaining calculator pages are briefly described here. There are also function definitions and other Mathboxes at the bottom of this page that are used in background calculations on other pages. These are essential to the tns file, so they should not be edited or changed. Some explorations may lead to the calculus concepts and use of these functions in the tns file.

Page 1.3



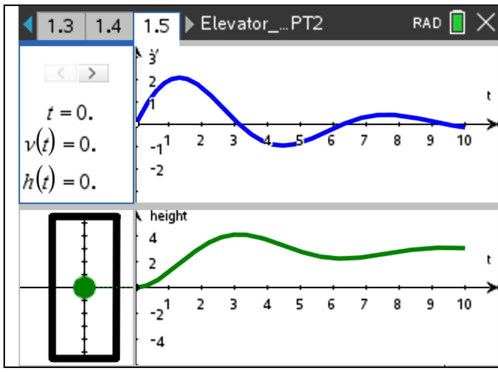
The graph of the velocity of the elevator for $0 \le t \le 10$ seconds is displayed in the bottom right pane. Values of t, $v(t)$, and $h(t)$ (in floors) are given in the top pane. The motion of the elevator is illustrated (in the left pane) as the value of t is changed, either by using the *clicker* (minimized slider) or by grabbing and dragging the point on the horizontal axis corresponding to t. At each value of t, there is an arrow drawn to the corresponding point on the velocity curve which represents the direction the elevator. The initial floor for the elevator is 0.

Page 1.4



The graph of the position of the elevator (floor) for $0 \le t \le 10$ (seconds) is displayed in the bottom right pane. It is also simultaneously illustrated in the left pane. Values of t, $v(t)$, and $h(t)$ are given in the top pane. The motion of the elevator is illustrated as the value of t is changed, either by using the clicker or by grabbing and dragging the point on the horizontal axis corresponding to t. The vertical axis in the bottom right pane for height corresponds to the height of the elevator in the left pane, as indicated by the dotted line.

Page 1.5



The graph of the velocity and the position of the elevator, for $0 \le t \le 10$ (seconds), are displayed simultaneously in the right two panes. The motion of the elevator is illustrated in the bottom left pane and the values of t, $v(t)$, and $h(t)$ are given in the top left pane. Also on this page, the motion of the elevator can be illustrated as the value of t is changed, here only by using the clicker.

Suggested Applications and Extensions

Elevator_Height_PT2.tns
Use the default velocity function. On page 1.3, observe the numerical values in the top pane and the motion of the particle in the bottom two panes as t increases from 0 to 10. The graph of the position of the object is given on page 1.4, and both graphs are displayed simultaneously on page 1.5. Use these graphs to answer the following questions.

1. Use the velocity function to explain when the elevator is moving up. Moving down.
2. Use the velocity function to explain when the elevator changes direction.
3. When is the elevator at its highest floor? Estimate the value of the velocity function at that time. What are the units for velocity?
4. Find the average velocity of the elevator over the interval $0 \le t \le 10$. Explain the meaning of this value using the graph of the position function.
5. Estimate the time(s) at which the acceleration of the elevator is 0. How are these times related to the graph of the velocity function?
6. Estimate the time at which the speed of the elevator is greatest.

7. Estimate a time t at which the (instantaneous) velocity is equal to the average velocity. Estimate the slope of the position function at this time. How does this value compare with the average velocity found in 4?
8. Estimate the first time $t_1 > 0$ at which the velocity is 0. Then estimate the area bounded above by the graph of the velocity, below by the t-axis, between $t = 0$ and $t = t_1$. How does this value compare with $s(t_1)$?

The following questions ask you to construct a velocity function that satisfies specific properties for $0 \le t \le 10$.

1. Construct a velocity function such that the elevator is always moving up but never reaches floor 5.
2. Construct a velocity function such that the elevator moves up to floor 5, then down to floor -5, and then ends at floor 0. Estimate the total distance traveled by the elevator.
3. Construct a velocity function such that the elevator moves up to floor 2, stops for 5 seconds, then moves back to floor 0.
4. Construct a velocity function such that the velocity is 0 at $t = 0, 1, 2, \ldots, 10$ and the elevator remains between floor -1 and 1. What is the average velocity over the interval $0 \le t \le 10$?
5. Construct a velocity function such that the displacement is equal to the total distance traveled.