Math Objectives

- Students will determine that the interior angle of a polygon and an exterior angle of a polygon form a linear pair (i.e., the two angles are supplementary).
- Students will determine that if one exterior angle is drawn at each vertex of a convex polygon, then the sum of the measures of those exterior angles is 360°.
- Students will determine a formula for the measure of one exterior angle of a regular polygon and use this to discover an alternative form for the formula that is typically used to calculate the measure of the interior angle of a regular polygon.

Vocabulary

- exterior angle of a polygon
- interior angle of a polygon
- regular polygon
- irregular polygon

About the Lesson

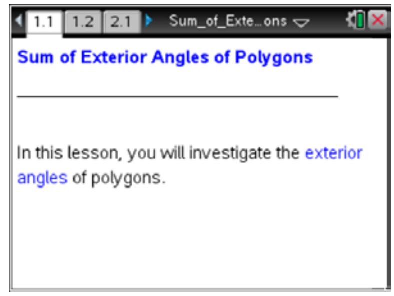
- This lesson involves moving an arrow along the side of regular and irregular polygons to form an exterior angle with the adjacent side of the polygon.
- As a result students will:
  - Discover that an interior and exterior angle of a polygon form a linear pair.
  - Determine that the sum of the measures of the exterior angles of **any** convex polygon is 360°.
  - Determine that an exterior angle of a regular *n*-gon must measure $\frac{360°}{n}$ and that the measure of the interior angle of a regular polygon can be found by the formula $180° - \frac{360°}{n}$.

TI-Nspire™ Navigator™

- Live Presenter
- Quick Poll
- Class Capture

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Materials:

Student Activity

- Sum_of_Exterior_Angles_of_Polygons_Student.PDF
- Sum_of_Exterior_Angles_of_Polygons_Student.DOC

TI-Nspire document

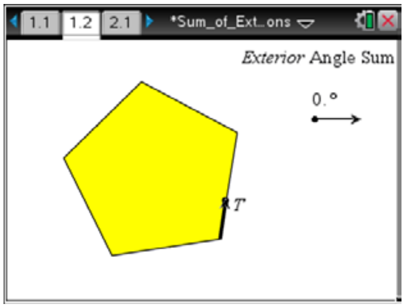
- Sum_of_Exterior_Angles_of_Polygons.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

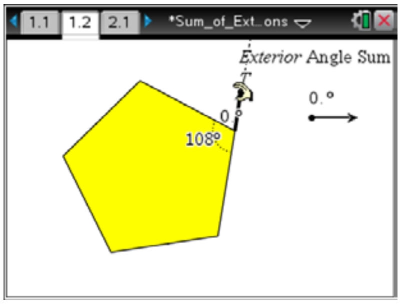
Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (👆) getting ready to grab the point. Press ctrl 👆 to grab the point and close the hand (✊).

Move to page 1.2.



1. The figure on page 1.2 is a regular pentagon.

Teacher Tip: You may want to demonstrate and have students follow you on the first rotation.



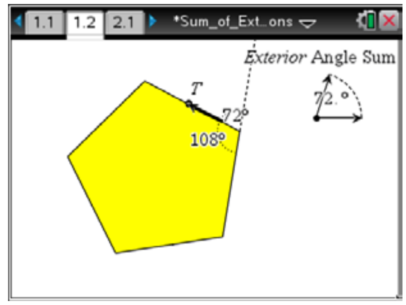
a. Move the arrow by dragging point *T* along the side of the pentagon. What appears when the arrow reaches the vertex?

<u>Answer:</u> A dotted ray appears. This ray forms an exterior angle with the next side of the polygon. The measure of the interior angle appears. The measure of the angle between the arrow and the dotted ray appears. Since the arrow is lined up with the dotted ray, this measure is 0°.

b. Make a conjecture about the number of degrees needed to rotate the arrow for it to line up with the next side of the pentagon. What mathematical relationship between the exterior and interior angles could help you determine the number of degrees needed to rotate the arrow?

<u>Answer:</u> Answers may vary. If students conjecture correctly, they will determine that they should rotate the vector 72°. Students should note that since the interior angle and the exterior angle at that vertex form a linear pair, they are supplementary and the sum of their measures is 180°. Therefore, you must rotate the arrow 72°.

c. Press esc. Test your conjecture by grabbing point *T* and rotating the arrow so that it "snaps" to the next side. What do you notice about the sum of the exterior angle and the adjacent interior angle?



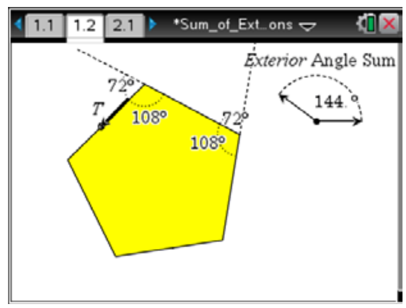
Answer: Their sum is 180°. They are supplementary angles.

Teacher Tip: After pressing esc, grab the arrow and rotate to the side. It should "snap" to the side, but this may take a little practice. If students have trouble, have them move the point a little closer to the next vertex and try again. The point will jump slightly when it snaps into place.

Tech Tip: To release point *T*, tap the white space outside of the polygon.

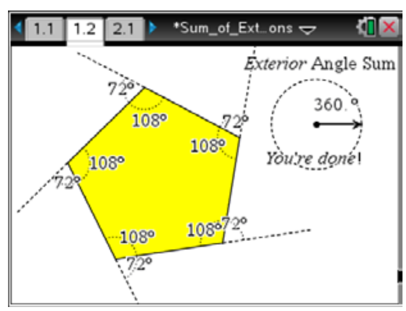
TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

d. Press esc. Move point *T* to the next vertex and complete the needed rotation for the arrow to line up with the next side of the polygon. Explain the new exterior angle sum.



Answer: The new sum is 144° because of another rotation of 72°.

e. Repeat steps a–d to continue moving and rotating the arrow until "You're done!" appears on the screen. How does the Exterior Angle Sum relate to the arrow's movement around the pentagon?

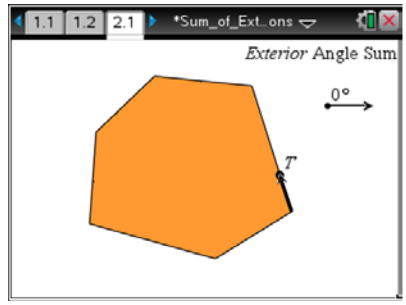


Answer: As the arrow moved around the pentagon, the Exterior Angle Sum increased by the amount that the arrow was rotated at each vertex. The sum vector rotated the same amount as the arrow on the pentagon, but the Exterior Angle Sum shows the total amount the arrow was rotated as it moved around the pentagon. The exterior angle sum is 360°.

Teacher Tip: Students may wish to undo their moves at some point. If so, have them use ctrl esc to undo their work, or close and reopen the document.

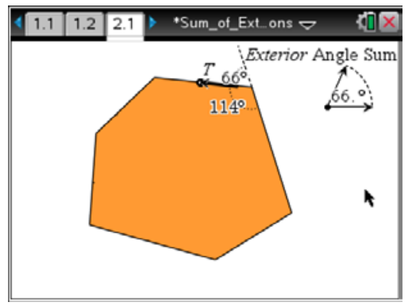
Move to page 2.1

2. The figure on page 2.1 is an irregular hexagon.



   a. Find the final exterior angle sum for the irregular hexagon. Before you begin, do you think the final exterior angle sum for the irregular hexagon will be the same as it was for the regular pentagon? Why or why not?

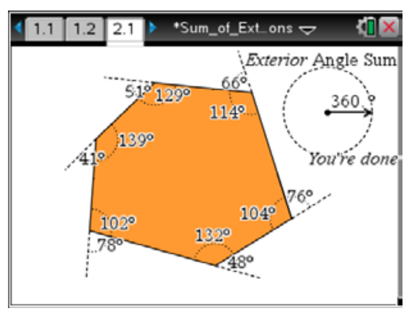
   <u>Answer:</u> Answers may vary. Students may assume correctly that the total will again be 360°, but many may incorrectly assume that the total will not be 360° because the shape is irregular.



   b. At each vertex, explain how you can determine the number of degrees needed to rotate the arrow for it to line up with the next side.

   <u>Answer:</u> The amount needed to rotate the arrow to the next side will always be 180° minus the measure of the interior angle at that vertex.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.



   c. Continue moving the arrow until the message "You're done!" appears on the screen. Observe the final exterior angle sum. How do the results compare to your expectations in part 2a?

   <u>Answer:</u> Answers may vary, but students should see that the total is indeed 360°. Depending on their answers to part 2a, they will make different comparisons.

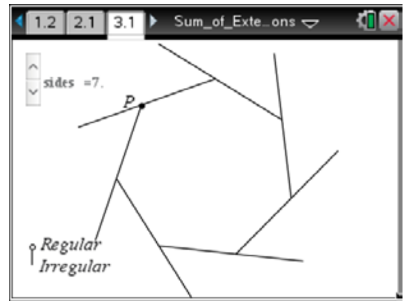
   Teacher Tip: Students will need to drop *T* and grab it again along each side until they have made it all the way around the polygon.

   d. Based on your findings, what do you think is true for the exterior angle sum of **any** regular or irregular polygon?

   <u>Answer:</u> The sum of the exterior angles of any regular or irregular polygon is 360°.

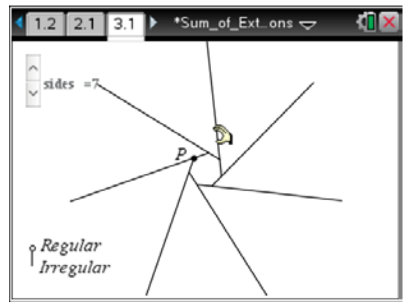
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

3. On page 3.1, there is a polygon with an exterior angle at each vertex.



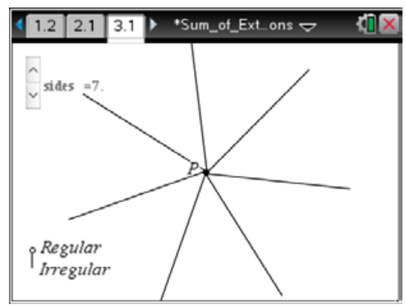
a. Drag point *P*. What do you observe about the exterior angles as the polygon changes size?

Answer: The exterior angles do not change.

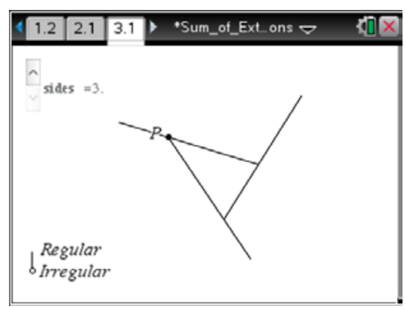


b. Drag point *P* so that the polygon shrinks to a point. What do you observe about the exterior angles?

Answer: They seem to meet at a point like radii of a circle (or spokes of a wheel).



c. Do some more experiments by moving point *P.* Click the up and down arrows at the top of the screen to change the number of sides of the polygon. Drag the open circle at the bottom of the screen to change the polygon from regular to irregular. What do you observe about the exterior angles?



Answer: When you drag point *P* so that the polygon shrinks to a point, it does not matter how many sides the polygon has, or whether it is regular or irregular. The exterior angles always meet at a point like radii of a circle, and thus their sum is always 360°.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 4 at the end of this lesson.

d. Based on your findings, what is the sum of the measures of the exterior angles of any polygon?

Answer: 360°

e. What is the sum of the exterior angles of a dodecagon (12-sided polygon)?

Answer: 360°

4. a. If you know the sum of the exterior angles of **any regular** *n*-gon, what formula could you use to determine the measure of one of its exterior angles?

Answer: The polygon is regular, so each interior angle must have the same measure. Therefore, each exterior angle would have the same measure. Since the sum of the exterior angles of the *n*-gon is 360°, the measure of one of the exterior angles is $\frac{360°}{n}$.

b. If you know the measure of an exterior angle of a regular *n*-gon, what formula could you use to determine the measure of one of its interior angles?

Answer: The interior angle would have to measure $180° - \frac{360°}{n}$.

c. How does the formula in part 4b relate to a common formula $\frac{(n-2)180}{n}$ that is given for the measure of the interior angle of a regular *n*-gon?

Answer: Students will need to use their algebra skills to show that the formulas are equivalent.

$$\frac{(n-2)180}{n} = \frac{180n - 360}{n} = \frac{180n}{n} - \frac{360}{n} = 180 - \frac{360}{n}$$

Teacher Tip: Possible extensions for students could include the exploration of the sum of the exterior angles of concave polygons.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- At any vertex of a polygon, the exterior angle and the interior angle are supplementary.
- If one exterior angle is drawn at each vertex of any convex polygon, the sum of the measures of these exterior angles is 360°.
- The measure of an exterior angle of a regular *n*-gon is $\frac{360^\circ}{n}$
- The measure of the interior angle of a regular *n*-gon is $180^\circ - \frac{360^\circ}{n}$.

TI-Nspire Navigator

Note 1

Question 1c, *Live Presenter*: Make a student the Live Presenter and have the student demonstrate how to pull point *T* out along the side of the polygon, release it, and then grab it again to rotate it in towards the next side.

Note 2

Question 2b, *Quick Poll*: One of the interior angles of a pentagon measures 118°. What is the measure of the exterior angle at the same vertex?

Answer: 62°

Note 3

Question 2d, *Quick Poll*: Four of the exterior angles of a pentagon measure 95°, 100°, 70°, and 65°. What is the measure of the fifth exterior angle?

Answer: 30°

Note 4

Question 3c, *Class Capture*: As students change the value of *n* and switch from regular to irregular on page 3.1, use Class Capture so that they can see results for a variety of *n*-gons.