



Math Objectives

- Students will learn review the concepts of a normal distribution and its characteristics, including but not limited to percentiles and area under a normal curve.
- Students will practice how to use the handheld commands **Normal Cdf** and **Inverse Normal**.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Percentile
- Normal Distribution
- Empirical Rule
- Probability
- Standard Deviation
- Mean

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content in Topic 4 Statistics and Probability:
 - 4.2:** (b) Histograms
(c) Percentiles
 - 4.3:** (a) Measures of central tendency (mean)
(c) Measures of dispersion (standard deviation and variance)
 - 4.9:** (a) The normal distribution and curve
(b) Properties of the normal distribution
(c) Diagrammatic representation, Normal probability calculations, Inverse normal calculations

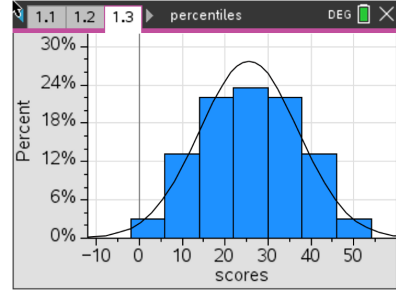
As a result, students will:

- Apply this information to real world situations.



TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>




Lesson Files:

Percentiles-Student-Nspire.pdf
 Percentiles-Student-Nspire.doc
Percentiles.tns
Percentiles_Soln.tns



- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

Activity Materials

Compatible TI Technologies:  TI-Nspire™ CX Handhelds,
 TI-Nspire™ Apps for iPad®,  TI-Nspire™ Software

The goal of this activity is for students to use the area to the left of a value in a normal distribution to find its percentile. The process will then be reversed to find the value for a given percentile. In doing so, students will learn how to use the **Normal CDF** and **Inverse Normal** commands on the handheld.

Teacher Tip: This activity is one where the *Percentiles.tns* file is key to fully understanding how the TI-Nspire CX II handheld can help students answer normal distributions both graphically (with histograms) and through the Normal commands mentioned above. Please take a moment to transfer the file to their calculators and refresh their memories on the sequence of entering values into the two Normal commands.

For this activity, students should be familiar with the normal distribution and its characteristics, specifically the empirical rule (68-95-99.7 rule). Percentiles divide data into 100 equal parts. For the sake of simplicity, round all percentiles to the nearest whole percent. The work for the activity will be done using the *Percentiles.tns* file. Move to page 1.2.

Problem 1 – Find the Percentile Given the Score

- (a) Using commands on your handheld, find the mean and standard deviation of the scores on page 1.3.

Solution: $\bar{x} = 25.66178 \dots$ and $s_x = 11.52623 \dots$
 (Since this is a sample we use s_x and not σ_x)

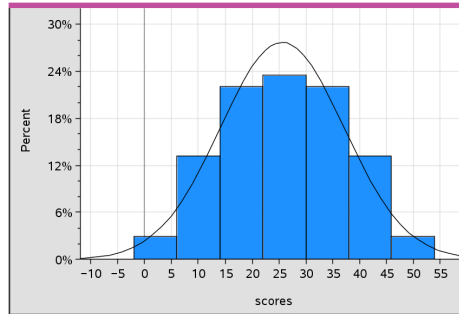
Teacher Tip: Show students how to find the mean and standard deviation while on the list and spreadsheet page using the **One-Variable Statistics** command. They should press **menu**, **Statistics**, **Stat Calculations**, **One-Variable Statistics**, then OK for one list, choose **scores** for X1 List,, and **b[]** for the result location.



(b) Create a histogram on page 1.4 and describe the distribution.

Possible Description: Approximately normal.

(c) Sketch your graph below.



Teacher Tip: Although students need to draw this histogram by hand, now would be a good time to demonstrate how to attain one on the handheld. First, from your data and statistics page, press **menu, Plot Type, Histogram**. Second, press **menu, Plot Properties, Histogram Properties, Histogram Scale, Percent**. To change the bin width and alignment, press **menu, Plot Properties, Histogram Properties, Bin Settings, Equal Bin Width**. Choose bin width 8 and alignment 6. To fit the new bins to the screen, press **menu, Window/Zoom, Zoom – Data**. Finally, if the students need help deciding if the histogram follows a normal distribution, they can press **menu, Analyze, Show Normal PDF**.

Move to page 1.5.

(d) Without calculating, and assuming the scores are normally distributed, find the percentile of the score if it is...

(note that hovering over a bin will display the percentage of scores in that bin)

(i) the mean.

Solution: 50%

(ii) one standard deviation above the mean.

Solution: 84%

(iii) one standard deviation below the mean.

Solution: 16%



Move to page 1.6.

- (e) Using the histogram and what you know about normal distributions, fill in the following table with your guess of the percentile for each score and the actual percentile for each score. Use the **Normal Cdf** command on page 1.7 to find the actual percentile for the scores in the table.

	33	50	26	12
Guesses	75%	95%	50%	10%
Actual	74%	98%	51%	12%

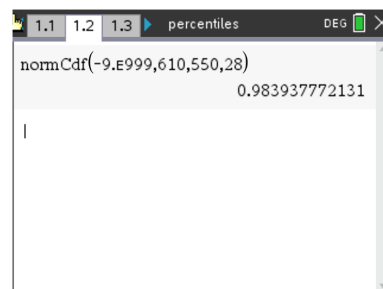
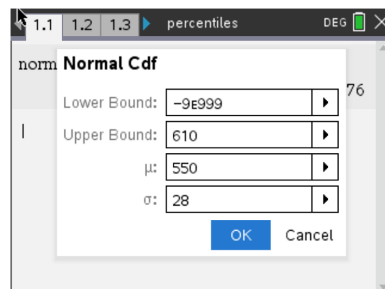
Solution: Answers for the guesses will vary, but a sample is given in the table above.

Teacher Tip: This would be a good time to define and discuss *percentile*. Stress that the percentile gives the proportion of the scores to the *left* of that score. Also, review the characteristics of a normal distribution and the empirical rule. Students are to determine what they think are the percentile scores on page 1.6. Discuss with the students how they determined their answers.

Move to page 1.8.

Problem 1 Practice

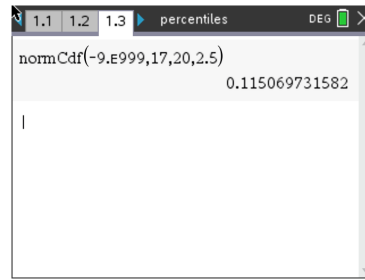
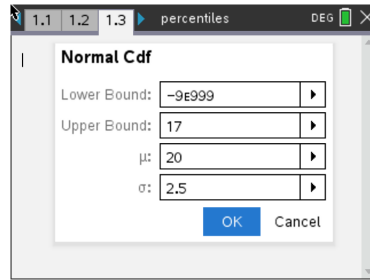
- (a) Assuming that the scores for the class were normally distributed and given that a student scored a 610 on a test with $\bar{x} = 550$ and $\sigma = 28$, find the percentile.



Solution: 98th percentile

Move to page 1.9.

- (b) Assuming that the scores for the class were normally distributed and given that a student scored a 17 on a test with $\bar{x} = 20$ and $\sigma = 2.5$, find the percentile.



Solution: 11th percentile

Move to page 2.1.

Problem 2 – Finding the Score Given the Percentile

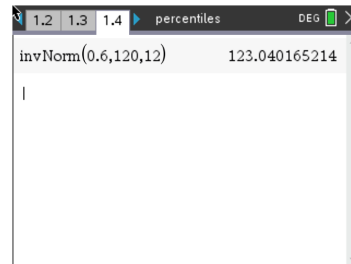
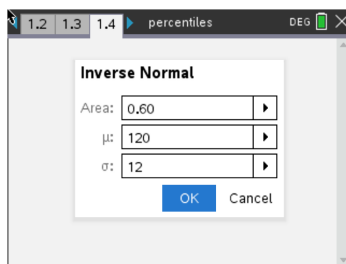
The scores on a test are normally distributed with a mean of 120 and a standard deviation of 12, or using the notation $N(120, 12^2)$. To reverse the process and find a score given its percentile, use the **Inverse Normal** command from the Distributions menu. When the pop up box appears, enter:

area, mean, standard deviation

The area in the pop up box refers to the area to the left of the given score, marked by the percentile.
**Remember that the normal distribution notation is $N(\mu, \sigma^2)$, where σ^2 represents the variance.

- (a) Estimate what you think the lowest score a student needs to be at least the 60th percentile. Explain your answer.

Solution: Answers will vary. The actual answer is 123.04.



Move to pages 2.2 and 2.3.

- (b) Verify your estimate from part (a) using the **Inverse Normal** command. Also use this command to help find the scores for the following percentiles:



	60 th	30 th	70 th	90 th
Guesses				
Actual	123.04	113.707	126.293	135.379

Solution: Answers will vary for the guesses.

Move to page 3.1.

Problem 3 – Practice

- (a) Helen took a test where $N(380, 42^2)$ and scored 465. Juan took a test where $N(65, 10^2)$ and scored 88. Determine which student is at the higher percentile.

Solution: Using the **Normal Cdf** command,
 Helen's percentile was 0.9785... or the 97.9th percentile
 Juan's percentile was 0.989276... or the 98.9th percentile
 Juan was in the higher percentile.

Move to page 3.2.

- (b) Ty scored lower than 14% of the rest of the students on a test with $N(200, 35^2)$. Estimate Ty's score.

Solution: Using the **Inverse Normal** command,
 Ty's score was about 237.811..., or 237

Move to page 3.3.

- (c) Find the score Shuang must get to be in the top 5% of the students taking a test with $N(325, 35^2)$.

Solution: Again, using the **Inverse Normal** command,
 Shuang's score is about 382.57..., or she had to get a score of at least 382



Further IB Applications

It is known through the company website that 43% of gecko heat lamp bulbs have a life of less than 60 hours and 89% have a life less than 65 hours. It can be assumed that heat lamp bulb life is modelled by the normal distribution $N(\mu, \sigma^2)$.

(a) Find the value of μ and the value of σ .

Solution: Using the idea of the area under a normal distribution curve is equal to the total probability in the normally distributed gecko heat lamp bulb scenario, first find the z-scores with the given percentages above:

$$P < 60 \text{ hours} : \text{InvNorm}(0.43, 0, 1) = -0.176$$

$$P < 65 \text{ hours} : \text{InvNorm}(0.89, 0, 1) = 1.23$$

Then, set up two equations using the z-score formula: $z = \frac{x - \mu}{\sigma}$

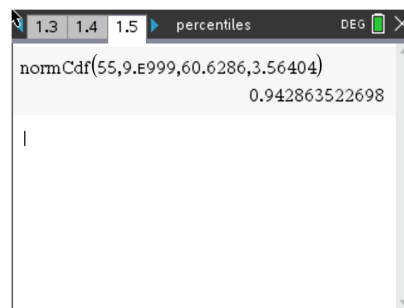
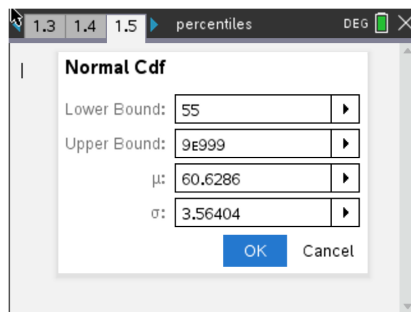
$$-0.176 = \frac{60 - \mu}{\sigma} \quad \text{and} \quad 1.23 = \frac{65 - \mu}{\sigma}$$

Solve the system of equations for μ and σ .

$$\mu = 60.6286 \quad \text{and} \quad \sigma = 3.56404$$

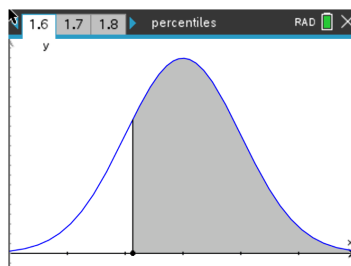
(b) Find the probability that a randomly selected bulb will have a life of at least 55 hours.

Solution: $P \geq 55$, using the Normal Cdf command:



$$= 0.943$$

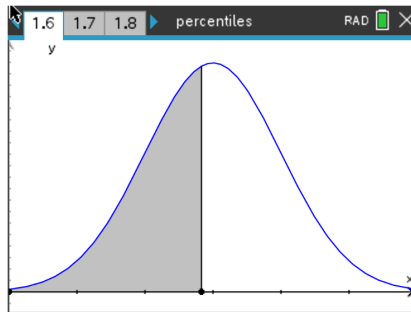
Having a visual representation of the normal distribution curve is helpful:





(c) Find the percentile for the bulb with 58 hours of life.

Solution: $P < 58$, $\text{normCdf}(-9E999, 60.6286, 3.56404) = 0.2303988\dots$
= 23rd percentile



TI-Nspire Navigator Opportunity: Quick Poll (Open Response)

Any part to any Problem in the activity would be a great way to quickly assess your student's understanding of percentiles.

Teacher Tip: Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review normal distributions, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*