EXPLORATIONS™

Activity 16

Objectives

- Examine functions defined by a definite integral
- Understand the foundation of the Fundamental Theorem of Calculus

Materials

- TI-84 Plus / TI-83 Plus

Accumulation Functions

Introduction

One of the greatest achievements in the history of mathematics was the discovery of the Fundamental Theorem of Calculus. In preparation for learning about this theorem, you must become familiar with a new type of function, in which a variable is a limit of integration.

You previously learned that an integral like the following is a real number:

$$\int_1^5 \frac{1}{t^2}dt$$

If the upper limit of integration were changed, say from 5 to 10, the integral would equal a different real number. You could make the upper limit any positive real number and get a different value for the integral. If you let the limit of integration be the independent variable, you can define a function such as

$$g(x) = \int_1^x \frac{1}{t^2}dt$$

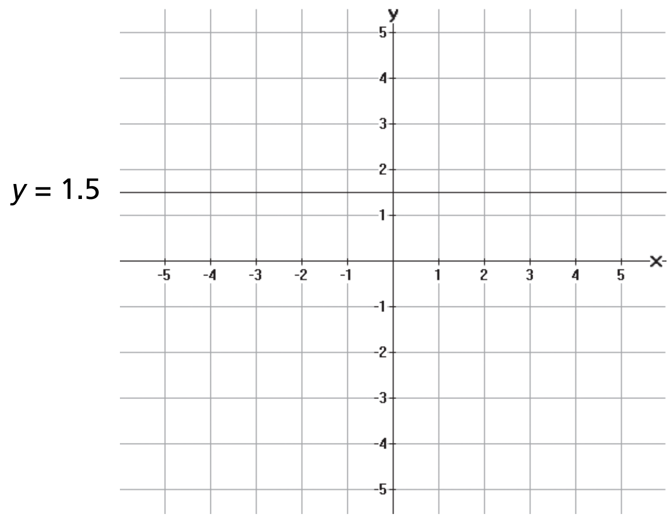
Such a function is an accumulation function because it measures the area accumulated under the graph of the integrand from the lower limit of integration up to a variable upper limit, *x*. In this activity, you will explore three accumulation functions.

Exploration

Consider the following function:

$$f_0(x) = \int_0^x 1.5\,dt$$

The function *f* measures the signed area under the graph of $y = 1.5$ from 0 to x.



1. When $x = 0$, the upper and lower limits of integration are the same.

   What is $f_0(0)$?

2. When $x = 1$, you are finding the area of a rectangle with a base 1 unit long and a height of 1.5 units.

   What is $f_0(1)$?

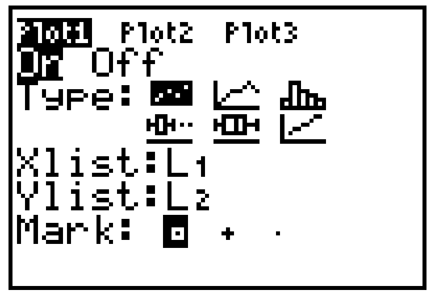
3. When $x = {}^-1$, the upper limit of integration is less than the lower limit. This makes the base of the rectangle negative.

   What is $f_0({}^-1)$?

4. Complete the table of values shown. Then enter the values of x from the table into the list **L1** and the values of $f_0(x)$ into the list **L2**.

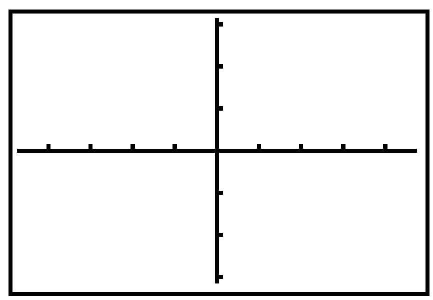
x	$f_0(x) = \int_0^x 1.5\,dt$
0	
1	
2	
3	
-1	
-2	
-3	

Set up a **STAT PLOT** as shown in this screenshot.



5. Select **9:ZoomStat** from the **ZOOM Menu** to view the scatter plot. Sketch the plot.

Now look at $f_1(x) = \int_1^x 1.5\,dt$.



6. Notice that this accumulation function has a different lower limit of integration. Here, area starts accumulating at a lower limit of 1 instead of 0. Notice that when $x = 1$, the upper and lower limits are the same.

What is $f_1(1)$?

7. When $x = 2$, you are finding the area of a rectangle with a base 1 unit long and a height of 1.5 units.

What is $f_1(2)$?

8. When $x = 0$, the upper limit of integration is less than the lower limit. This makes the base of the rectangle negative.

What is $f_1(0)$?

9. Complete the table of values.

x	$f_1(x) = \int_1^x 1.5dt$
0	
1	
2	
3	
-1	
-2	
-3	

10. Graph the ordered pairs (x, $f_1(x)$) on the same set of axes used for Question **5**.

11. Finally, look at the accumulation function:

$$f_{-1}(x) = \int_{-1}^x 1.5dt$$

Fill in the table of values.

x	$f_{-1}(x) = \int_{-1}^x 1.5dt$
0	
1	
2	
3	
-1	
-2	
-3	

12. Graph the ordered pairs (x, $f_{-1}(x)$) on the same set of axes you used for Question **5**.

13. What do your three graphs have in common?