About the Lesson

In this activity, students will explore data graphically and algebraically. As a result, students will:

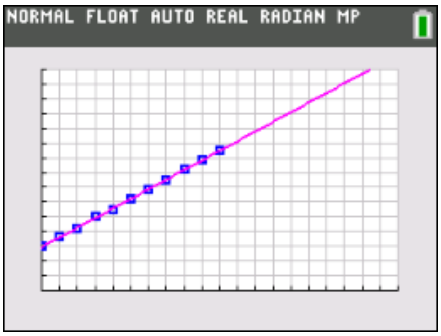
- Graph a scatterplot of data and determine a linear function that describes the data.
- Analyze a scatterplot to determine qualitative and quantitative trends in data.
- Determine the equation of a line of best fit to analyze and make predictions about data.
- Interpret the real-world significance of the slope and *y*-intercept of a linear model.

Vocabulary

- line of best fit
- linear regression
- slope
- *y*-intercept

Teacher Preparation and Notes

- Students can plot the graph by hand, use technology to confirm, and analyze the data graphically and algebraically.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-84 Plus family with the latest TI-84 Plus operating system (2.55MP) featuring MathPrint™ functionality. Slight variations to these directions given within may be required if using other calculator models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- Marble_Experiment_Student.pdf
- Marble_Experiment_Student.doc

Tech Tip: On the TI-84 Plus C Silver Edition, turn on the GridLine by pressing 2nd ZOOM to change the FORMAT settings. Note that the GridLine feature is unique to the TI-84 Plus C Silver Edition.

Camilla's math class was performing an experiment to see if they could come up with a relationship connecting the number of marbles dropped in a clear glass cylinder and the height of the water after each drop.

Camilla's group started with a 14-inch tall cylinder filled with 3 inches of water. As one member dropped a marble in, another measured the height of the water, and another recorded it in a table. Their data table is shown to the right.

Number of Marbles	Height of Water (in inches)
0	3
1	$3\frac{5}{8}$
2	4.25
3	5
4	5.5
5	6.25
6	$6\frac{7}{8}$
7	7.5
8	$8\frac{1}{4}$
9	$8\frac{7}{8}$
10	9.5

Answer the following questions based on their data.

1. What kind of mathematical model would best fit the data in this table?

   Answer: A linear model. After students plot the data they will be able to see more clearly that the data are linear.

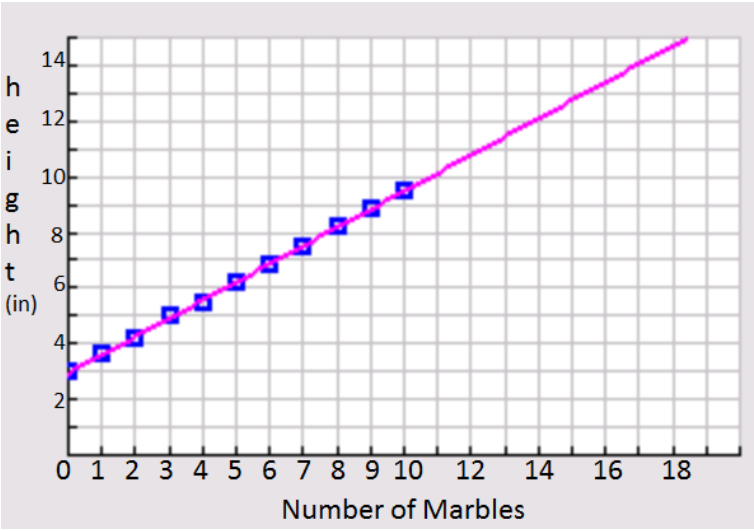
2. Plot the data below. Label the axes and provide the scale. Confirm by plotting the data on your graphing calculator with a similar viewing window. Draw and write an equation that best fits the data. How did you determine this equation?

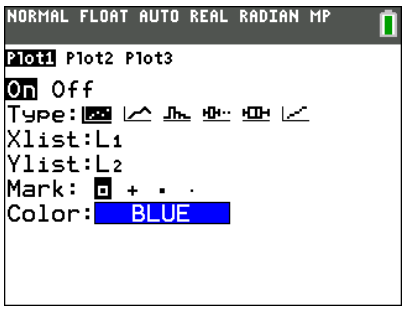
   Sample Answer: Students can plot the points by hand and estimate the slope and *y*-intercept.

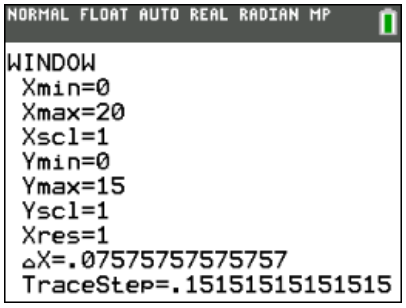
   Their equation should be close to $y = 0.65x + 2.97$.

Teacher Tip: Encourage students compare the line of best fit they found by plotting the points by hand and the line of best fit found on their graphing calculator from the regression line as described below.

With real-world data students should not use the actual values of data points to find the slope of the line of best fit. Instead, after graphing, they should show work using values taken from two points on the line of best fit. For example, from the graph below (3,5) and (17,14) appear to be on the line. This gives a slope of (14-5)/(17-3) = 9/14 which is close to what the regression equation gives.



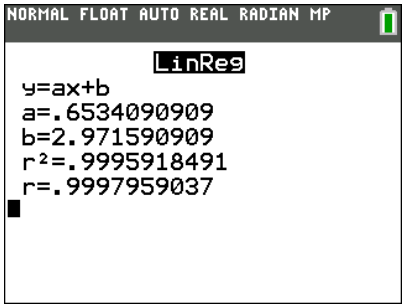




Tech Tip: To plot the data on the TI-84, have students first enter the data in L_1 and L_2 by pressing [STAT] [ENTER]. Students can input mixed fractions by pressing [ALPHA] [F1]. Then, have students press [2nd] [Y=] for [STAT PLOT] and turn on Plot1. Students should choose the Xlist to be L_1 ([2nd] [1]) and Ylist to be L_2. Have students press [WINDOW] and set the domain and range based on the data. They should then press [GRAPH] to see the plotted data.

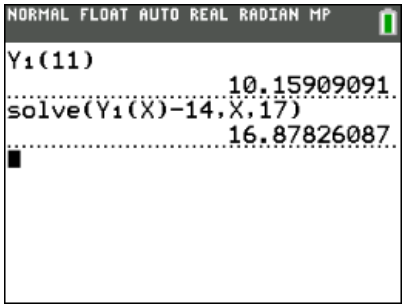
To use the TI-84 to calculate the line of best fit, have students select the linear regression by pressing [STAT], move the cursor to the CALC menu, and select **LinReg(ax+b)**. Again, have students enter L_1 as the Xlist and L_2 as the Ylist. Students should store the regression equation in $\mathbf{Y_1}$ using [ALPHA] [F4] and press [GRAPH].

3. If an eleventh marble is placed in the cylinder and the water's height is measured, use your equation to predict the height of the water.



Sample Answer: Students can compare the answer from the regression equation, $Y_1(11) = 10.16$ inches, with the plotted graph where the height appears to be 10 in. (Note that if students use the rounded values from the regression equation expressed in question 2, they will obtain a height of 10.12 in.) Students can also press [TRACE], select the line, and enter 11 to see this height graphically on the TI-84.

4. How many marbles can be dropped into the cylinder so that the cylinder is as full as possible without overflowing?



Answer: Since the height of the cylinder is 14 inches, students are to solve $Y_1(x) = 14$, where x is the number of marbles and Y_1 is the height. Solving the regression equation $0.65x + 2.97 = 14$ for x results in $x = 16.87$ marbles. If 17 marbles are put in, then the water will overflow. So 16 marbles are the maximum number of marbles without overflowing.

Teacher Tip: Attending to precision is done in these problems as students include units to communicate the meaning of the slope and *y*-intercept, and it is also done when they consider how many digits to write down. Graphically, students can only display one decimal of precision. The data suggests that the height was measured to at least two decimals of precision although this was not always recorded. The number of marbles is exact precision. The linear regression or line of best fit is a model. The model is continuous, but the real world situation is discrete. There are only integer values of marbles.

5. From the line of best fit equation explain what the rate of change is and what it means in the context of this experiment. Be sure to use correct units in your description.

Answer: The rate of change is the slope, 0.65 inches/marble. For every marble dropped in the glass the height of the water raises on average 0.65 inches.

6. What is the *y*-intercept of the graph and what does it represent?

Answer: The *y*-intercept is approximately 3 inches. This is the height of the water when there are no marbles in the glass.

7. If the initial water level were 5 inches, how would the graph be different? In what ways would the graph be the same?

Answer: The only change would be the *y*-intercept. The rate of change, or slope, will be the same, but the graph will be vertically shifted up by 2 inches.

Teacher Tip: Student can collect their own data if you have marbles and a clear cylindrical cup or graduated cylinder. Have them model the data and use the equation to predict the height of the water for more marbles than tested in their experiment. Be sure students can explain the physical meaning the rate of change and *y*-intercept.