Making Limits Exist

Student Activity

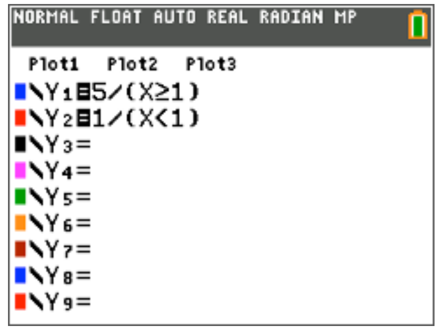
Name ____________________

Class ____________________

Problem 1 – Linear Piecewise Function

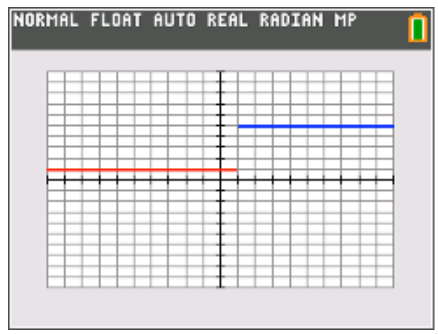
Graph the piecewise function $f(x)=\begin{cases} a, x \geq 1 \\ 1, x < 1 \end{cases}$ where *a* is a constant.

Step 1: Press [y=] and enter the two equations you see at the right into your device. The inequality symbols can be found by pressing [2nd][math]. Note that we have begun with an *a*-value of 5.

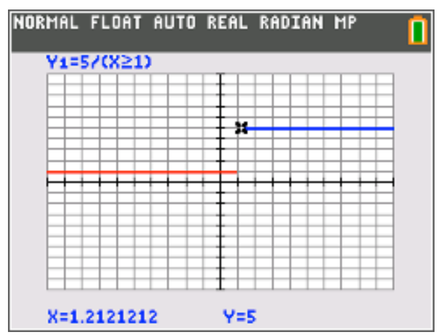


Note: To set the domain for piecewise functions, each piece must be entered into its own equation line and be divided by its restricted domain.

Step 2: Set the viewing window to standard by pressing [zoom] and selecting **ZStandard**.



Step 3: Press [trace] and use the left/right arrow keys to move along the domain of each piece. Press the up/down arrows to move between the pieces.



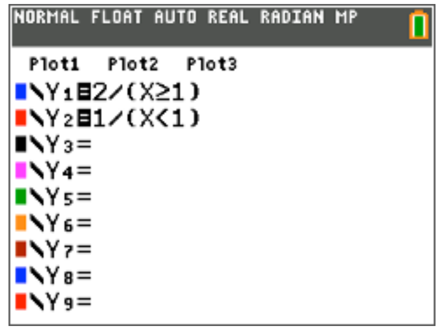
1. Graphically, what do the following one-sided limits appear to be?

$$f(x)=\begin{cases} 5, x \geq 1 \\ 1, x < 1 \end{cases}$$

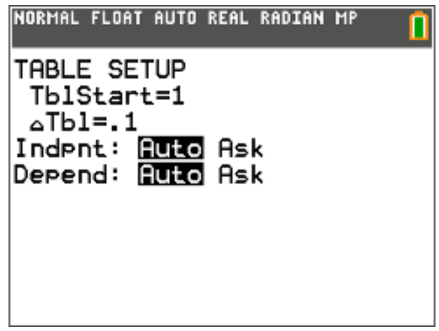
a. $\lim_{x \to 1^-} f(x) \approx$ _____

b. $\lim_{x \to 1^+} f(x) \approx$ _____

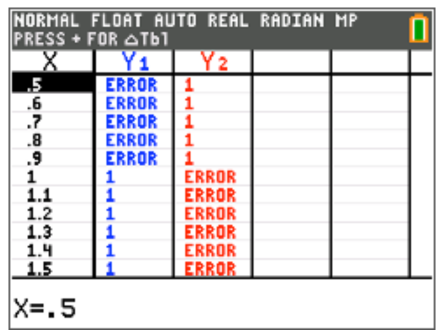
Step 4: Try other values for *a* in our piecewise function $f(x)=\begin{cases} a, x \geq 1 \\ 1, x < 1 \end{cases}$ to determine what *a*-value makes $\lim\limits_{x\to 1} f(x)$ exist. Remember, **Y1** is the function that has the *a*-value we are changing. In the screen to the right, *a* has been changed to 2. After changing the *a*-value, press [graph] to see the resulting changes in the graph. Try different values for *a*. Graph it to see if *f(x)* appears continuous.



Step 5: Check your answer numerically to determine if your *a*-value is correct. Set up the table by pressing [2nd] [window] and changing the settings to those on the right.



Step 6: Now, press [2nd] [graph] to view your table. Use the up and down arrows to move through the table. The table will show ERROR for any *x*-value that is not in the domain of the **Y2** or **Y2**.



2. After checking graphically and numerically, what value of *a* resulted in *f*(*x*) being continuous?

Problem 2 – Linear and Quadratic Piecewise Function

Repeat the steps from earlier for the function $g(x)=\begin{cases} a\cdot x^2, x \geq 1 \\ x+2, x < 1 \end{cases}$ starting with an *a*-value of 5.

3. Graphically and numerically, what do the following one-sided limits appear to be?

$$g(x)=\begin{cases} 5\cdot x^2, x \geq 1 \\ x+2, x < 1 \end{cases}$$

a. $\lim\limits_{x\to 1^-} g(x) \approx$ _____

b. $\lim\limits_{x\to 1^+} g(x) \approx$ _____

4. a. After checking graphically and numerically, what value of *a* resulted in g(x) being continuous?

b. Show calculations of the left hand limit and the right hand limit to verify that your value for *a* makes the limit exist.

Problem 3 – Trigonometric Piecewise Function

Repeat the steps from earlier for the function $h(x)=\begin{cases} a+3\sin\left(\left(x-4\right)\frac{\pi}{2}\right), x\geq 2 \\ 2\sin\left(\left(x-1\right)\frac{\pi}{2}\right), \quad x<2 \end{cases}$ starting with an *a*-value of 5.

5. Graphically and numerically, what do the following one-sided limits appear to be?

$$h(x)=\begin{cases} 5+3\sin\left(\left(x-4\right)\frac{\pi}{2}\right), x\geq 2 \\ 2\sin\left(\left(x-1\right)\frac{\pi}{2}\right), \quad x<2 \end{cases}$$

6. a. After checking graphically, and numerically, what value of *a* resulted in *h*(x) being continuous?

b. Show calculations of the left-hand limit and the right-hand limit to verify that your value for *a* makes the limit exist.