

## Geometric Sequences & Series

ID: 8682

 Time required  
 30 minutes

### Activity Overview

*Students will examine geometric sequences and series in Sequence mode, relate geometric sequences to their explicit forms, find the partial sums of a sequence in a table, and determine if a geometric series is converging.*

### Topic: Sequences, Series, & Functions

- *Calculate the  $n$ th term of a sequence defined by an algebraic expression.*

### Teacher Preparation and Notes

- *This activity is designed to be used in an Algebra 2 classroom.*
- *Students should begin this activity knowing that a sequence is an ordered list of numbers that follows a pattern and that a series is an indicated sum of a sequence. For example, 1, 2, 3, 4 is a sequence and  $1 + 2 + 3 + 4$  is a series.*
- *Prior to beginning the activity, students should clear out any functions from the Y= screen and any data from the Lists.*
- ***To download the student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter "8682" in the keyword search box.***

### Associated Materials

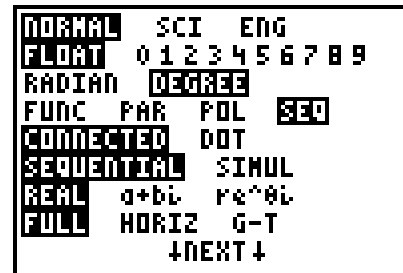
- *GeometricSequencesSeries\_Student.doc*

### Suggested Related Activities

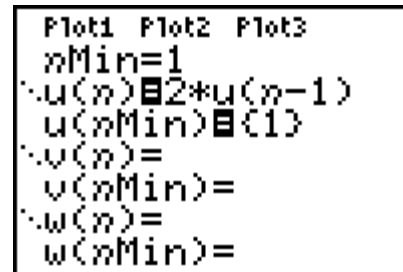
*To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the keyword search box.*

- *Infinite Geometric Series (TI-84 Plus family) — 11065*
- *Arithmetic Sequences & Series (TI-84 Plus family) — 8642*
- *Sums of Sequences (TI-84 Plus family) — 11134*

First students will need to change the **MODE** to **SEQ** as shown at the right.



When they press **Y=**, they should notice that the screen looks vastly different than when it is in function mode.



Students will first consider the sequence where  $a_1 = 1$  and  $a_n = 2 * a_{n-1}$ .

Press **2nd** [**u**] to enter **u**.

Press **X,T,θ,n** to enter **n**.

**Note:** The braces surrounding 1 for  $u(nMin)$  will appear after they press enter if they choose not to type them.

Looking at the table, students should see that the numbers appear to be doubling and that each term is changing at an increasing rate.

n	u(n)	
1	1	
2	2	
3	4	
4	8	
5	16	
6	32	
7	64	

n=1

The function that produces the same table is  $f(x) = 2^{x-1}$ .

**If using Mathprint OS:**

When students view the table, the line **Press + for ΔTbl** will appear at the bottom of the screen. Have them ignore this command because altering the step size of  $n$  will result in an error.

n	u(n)	
0	ERROR	
1	1	
2	2	
3	4	
4	8	
5	16	
6	32	

Press + for ΔTbl

Next students will find the partial sums of the terms in the sequence **u(n)**.

Press **2nd** [**v**] to enter **v**.

When they view the table, students should see that **v(n)** adds up the terms in **u(n)**. The sum of the first six terms is 63.

n	u(n)	v(n)
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127

n=1

When students scroll down in the table to see more terms in the sequence and series, they should notice that both values appear to be getting larger and larger and do not appear to be stabilizing.

$n$	$u(n)$	$v(n)$
10	512	1023
11	1024	2047
12	2048	4095
13	4096	8191
14	8192	16383
15	16384	32767
16	32768	65535

$n=16$

Now students will consider the sequence where  $a_1 = 5$  and  $a_n = 0.1 * a_{n-1}$ .

When they view the table of values, students should see that the value of each term is getting smaller and smaller at a non-constant rate. The function that produces the same table is  $f(x) = 5 * 0.1^{x-1}$ .

$n$	$u(n)$	
1	5	
2	.5	
3	.05	
4	.005	
5	5E-4	
6	5E-5	
7	5E-6	

$n=1$

**If using Mathprint OS:**

Students can compare the graphs of  $a_n = 0.1 * a_{n-1}$  and  $f(x) = 5 * 0.1^{x-1}$ . Have students press **MODE** and change the fourth line back to **FUNC**. Then they can press **Y=** and enter the function. When they press **^**, the cursor will move to the exponent position.

Students should enter the value of the exponent and then press **▸** to move out of the exponent position.

Plot1	Plot2	Plot3
Y1=5*0.1 <sup>X-1</sup>		
Y2=		
Y3=		
Y4=		
Y5=		

Next students find the sum of the terms in the sequence  $u(n)$  using  $v(n)$ . Once again students should see that  $v(n)$  adds up the terms in  $u(n)$ , but this time the sum appears to be converging to one value, about 5.6.

$n$	$u(n)$	$v(n)$
1	5	5
2	.5	5.5
3	.05	5.55
4	.005	5.555
5	5E-4	5.5555
6	5E-5	5.55555
7	5E-6	5.555555

$n=1$

Students are to turn off  $u(n)$  so that they can graph only the series. They should see that as  $n$  increases the graph looks like a horizontal line and it looks like the sum will never reach 5.6.

Encourage students to try other series to determine when it converges or diverges.

y=v(x-1)+0.1*u(x-1)	
. . . . . X .	
.	
.	
X=9	Y=5.5555556