ACTIVITY 8

Hooke's Law: The Rest of the Story

Math Objectives:

- Graph scatter plots
- Analyze and graph linear functions
- Calculate and apply slope
- Discuss positive versus negative slopes

Materials: (Per Group)

- TI-83/TI-84 Plus Family
- Data from Activity 7: Hooke's Law

OVERVIEW

In Activity 7, Hooke's Law was applied to the linear behavior of a spring as it is pulled downward. The stretch on the spring was directly proportional to the number of candies that was put in the cup. The data collected was the distance the cup moved towards the floor as the cup was filled with candy. To save time, the data collected in Activity 7 will be used to generate a new set of data. The new data will display the distance from the top of the meter stick to the bottom of the container vs. number of candies. This measures the stretch of the spring instead of the distance between the cup and the floor. A linear function will be generated from this new data, but this line will have a positive slope since the distance the spring stretches will increase as the number of candies increases. This activity is used to help students acquire a more thorough understanding of the meaning of slope. You will create a second linear model and then apply the properties of a linear function to develop a model for this distance. As in Activity 7, you will analyze the data and interpret the meaning of the slope. The slope is then used to develop an equation of the line of best fit.

NOTE A Slinky® works well for the spring and M&Ms® work well for the candies.

SETUP

1. This activity does not require you to set up and conduct new data collection trials. The students may need a diagram to see the difference between these two activities and to understand the logic of creating new data lists. Start by going to the Stat List Editor (Press [STAT] [ENTER]) and have the three lists from Activity 7 displayed next to each other. **See Figure 1.**

HSEQ	HOOKS	HDIF 1
0	1.4962	-.0938
3	1.4024	-.0877
6	1.3147	-.0729
9	1.2418	-.085
12	1.1568	-.0751
15	1.0817	-.0803
18	1.0014	-.0797

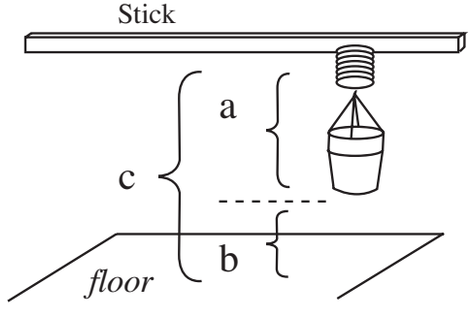
HSEQ(1) =0

Figure 1

NOTE *For help with the list display, see Appendix E, sections 19-23.*

DATA COLLECTION

1. From Activity 7, recall the distance from the stick to the floor. Lead the students in a discussion that will help them develop a formula for the distance from the stick to the bottom of the cup as candies/almonds are added to the cup. Use a sketch like the one shown here to demonstrate that the distance from the stick to the bottom of the cup (a) equals the total distance (c) minus the distance from the cup to the floor (b). If students struggle with this, substitute values for the variables. If the cup is 20 inches from the floor (b) and the stick is 30 inches from the floor (c), then the cup would be 10 inches from the stick ($c - b = a$).



2. Review which data is in which list. **HSEQ** is the number of candies/almonds added to the cup, **HOOKS** is the distance from the cup to the floor/CBR 2, and **HDIF** is the difference between any two consecutive entries in the **HOOKS** list.

3. Examine the measurements in the lists from Activity 7 and discuss their units. The CBR 2 was set to measure this data in feet. Convert the distance from the stick to the floor from inches into feet. For example, 30 inches = $\frac{30}{12}$ = 2.5 feet.

4. Have students convert their measurement from the stick to the floor to feet and round to the nearest hundredth. In this example, the distance was 29.5 inches. Converted to feet and rounded to the nearest hundredth, it equals 2.46 feet. Insert an empty list behind the three lists used in Activity 7. In this example, **L3** is the new list. Be sure **L3** is highlighted and define it to be the total length (2.46) minus the measurements in the **HOOKS** list. **See Figure 2.**

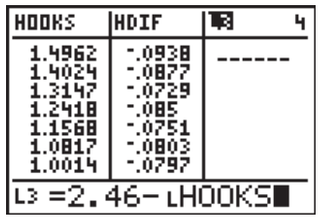


Figure 2

★ **NOTE** *For help working with lists, see Appendix B.*

5. After entering the formula, press [ENTER] to execute the command. The list will fill in with the results. **See Figure 3.**

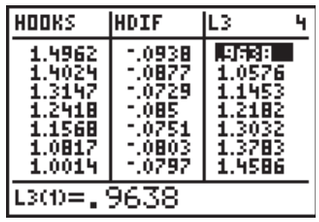


Figure 3

6. Set up a scatter plot for the distance from the meter stick to the bottom of the container vs. number of candies. Turn off any previous scatter plot or graph entered into **Y1** and/or **Y2**. To access a named list, position the cursor behind **Xlist:** and press [2nd] [STAT] to access the [LIST] menu. Scroll through the choices until you highlight the list you want and then press [ENTER]. Repeat the process for the **Ylist. See Figure 4.**

Figure 4

7. Press [ZOOM], and then select **9:ZoomStat** to set a window with appropriate parameters for the data. **See Figure 5.**

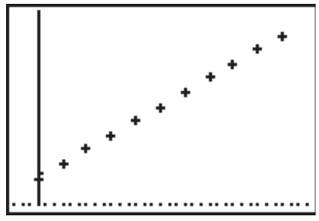


Figure 5

8. Press [TRACE], and then look in the upper left corner to confirm the lists that were used. **See Figure 6.**

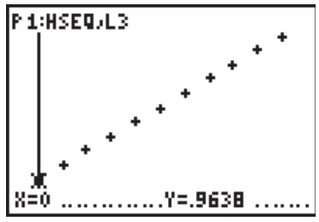


Figure 6

9. Repeat the procedure used in Activity 7 and use the differences to estimate the rate of change. Define **L4** to be the difference of the distances in **L3**. **See Figure 7.**

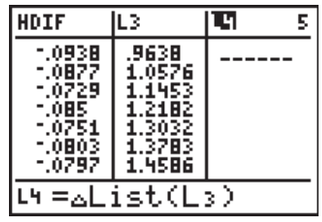


Figure 7

10. Be sure to have students draw the comparison between these differences and the **HDIF** list. Point out that the values in **L4** represent the actual amount of movement of the cup. **See Figure 8.**

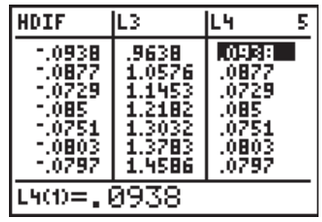


Figure 8

DATA ANALYSIS

1. Students can repeat the procedure they used in Activity 7 to identify the slope and **Y**-intercept for the line of best fit. Remind them that because the change in the **X**-values is always three, they will need to divide the differences in **L4** by three. This quotient will result in finding the slopes for the individual segments between any two consecutive points listed.

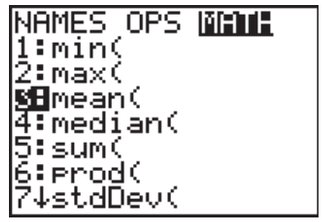


Figure 9

2. In Activity 7, a new list was created of these slopes and the mean of the list was calculated. This process can be accomplished on the home screen. To do this, press 2nd MODE to access [QUIT] and return to the home screen. Then press 2nd STAT to access the [LIST] menu and arrow over until **MATH** is highlighted. Select **3:mean(** from the list. **See Figure 9.** Enter **L4/3**) STO▶ ALPHA **N** in the home screen. Store the value in **N**, so it can be retrieved when needed. Press ENTER. **See Figure 10.**

Figure 10

3. Help your students understand that the average of this difference list divided by three will serve as a close approximation to the slope of the line of best fit for all points in the plot.

4. Next, identify the **Y**-intercept. Return to the graph and trace to the first data point. **See Figure 11.**

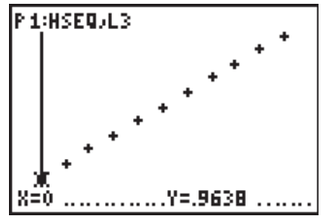


Figure 11

5. Press 2nd MODE to access [QUIT] and return to the home screen. Take the **Y**-value from the first data point and store it in the **C** variable. The keystrokes are as follows: ALPHA **Y** STO▶ ALPHA **C** ENTER. **See Figure 12.**

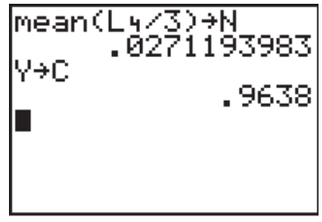


Figure 12

6. Using the **Y**-intercept and the slope, enter an equation that approximates your trend line. Next, look at the graph to see if the equation is a close fit to the data points. Go to the [Y=] window and press [ALPHA] **N** [X,T,Θ,n] **+** [ALPHA] **C** to type in **NX+C** beside **Y1. See Figure 13.**

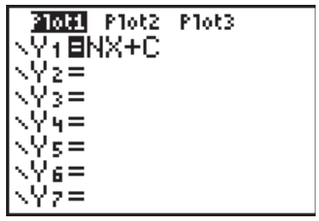


Figure 13

7. Press [GRAPH] to see how closely your line fits the points. Press [TRACE] and use the left and right arrow keys to scroll through the data points. **See Figure 14.**

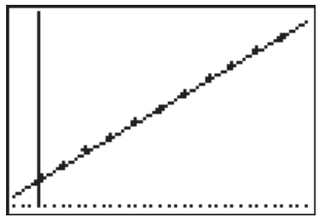


Figure 14

8. Turn on **Plot2** and graph the data from Activity 7 on the same coordinate plane. **See Figure 15.**

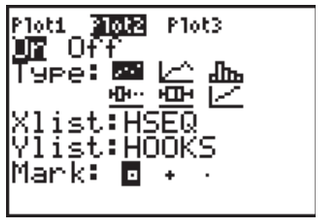


Figure 15

9. If your students have not cleared the calculator's memory from Activity 7, go to the [Y=] window and press [ALPHA] **M** [X,T,Θ,n] **+** [ALPHA] **B** to type in **MX+B** beside **Y2**. If the memory has been cleared, have the students retrieve these values from their Activity 7 worksheet. **See Figure 16.**

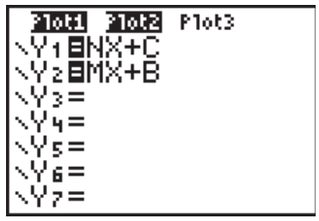


Figure 16

10. Press [GRAPH] to see both equations graphed along with the data points from the [STAT PLOT] menu. Lead students in a discussion about the similarity in the amount of slant in the two lines even though the lines are going in different directions. **See Figure 17.**

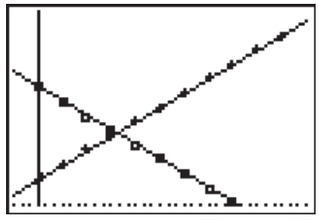


Figure 17

11. Return to the home screen and press [ALPHA] **N** [ENTER], and then [ALPHA] **M** [ENTER] to recall **N** and **M**. If **M** has been cleared from the calculator's memory, refer to the Activity 7 worksheet to retrieve it. Make sure the students realize why these numbers are the same but have opposite signs. Use language that will help build their understanding of slope as the change in the distance per the change in the candies. **See Figure 18.**

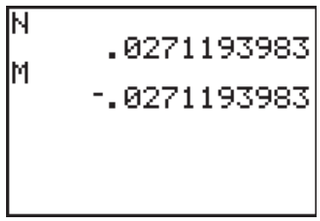


Figure 18

12. Go into the [WINDOW] screen and adjust the **Ymin** value to a small negative number so the **X**-axis will be seen in the graph screen. **See Figure 19.**

13. If the **X**-range is adjusted to be a multiple of 47, you will generate "friendly" numbers as the graph is traced. **See Figure 19.**

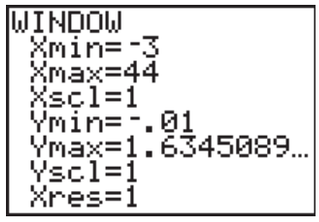


Figure 19

14. An extension activity is to use the trace feature to examine the sum of the **Y**-values (distances) for any given **X**-value (number of candies). **See Figures 20a–b.**

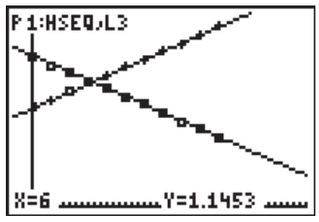


Figure 20a

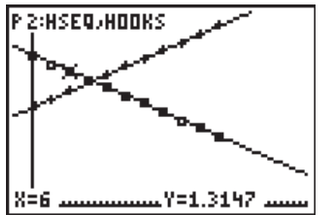


Figure 20b

15. Scroll to use a second **X**-value. Be sure to pay attention to what is written in the upper left corner of the screen. Check the actual measurements from **P1** and **P2**. **See Figures 21a–b.**

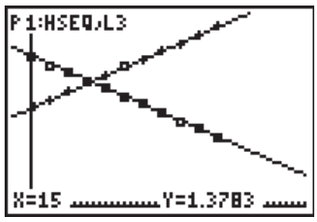


Figure 21a

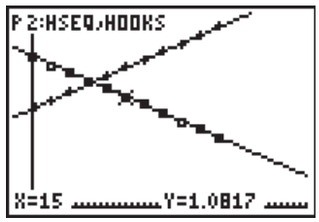


Figure 21b

16. Hopefully, your students will recognize that the sum of the **Y**-values is the total distance from the stick to the floor. Help them keep track of what set of data is in each list and what lists are the coordinates of each plot. **P1** is the distance from the stick to the bottom of the cup and **P2** is the distance from the cup to the floor. For any given number of candies (an **X**-value), the sum of the two distances equals the total distance. From the diagram at the beginning of this activity, $a + b = c$.

Figure 22a

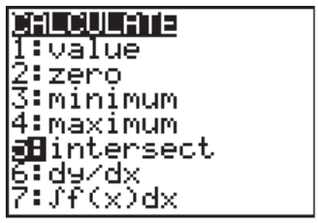
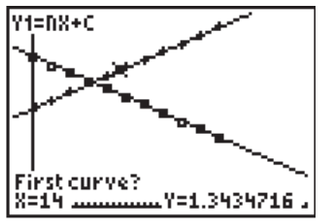


Figure 22b



17. Use the calculator to identify the point of intersection. To do this, press [2nd] [TRACE] to access the [CALC] menu and select **5:intersect**. **See Figure 22a.**

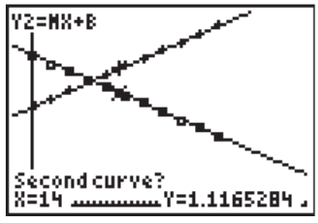


Figure 22c

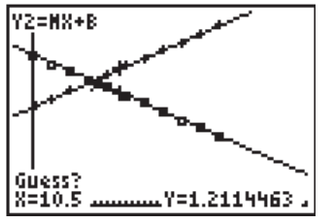


Figure 22d

18. The cursor is on **Y1**. Press [ENTER] to identify it as the **First curve?**. **See Figure 22b**. The cursor will jump to **Y2**. Press [ENTER] again to identify it as the **Second curve?. See Figure 22c**. When asked for a **Guess?**, scroll until you are close to the point of intersection and press [ENTER] again. **See Figure 22d.**

19. The coordinates of the point of intersection are displayed at the bottom of the screen. Help your students interpret this as the point where the bottom of the cup was the same distance from the floor as it was from the top of the stick. **See Figure 23.**

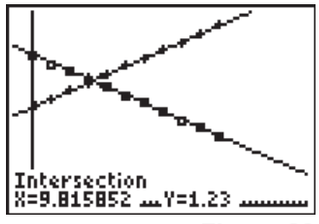


Figure 23

EXTENSION

1. Another extension is to measure the height of the cup and create a new list that would stand for how far the top of the cup was from the stick. This is found by subtracting the height of the cup from the distances in **L3**. Go back to the Stat Editor and define **L5** to be **L3 – ht of the cup** (in feet). In this sample, the cup was 3 inches or $\frac{3}{12}$ = 0.25 ft. So **L5=L3 –.25**. **See Figure 24.**

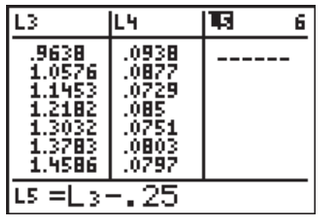


Figure 24

2. Press ENTER to fill in the list. **See Figure 25.**

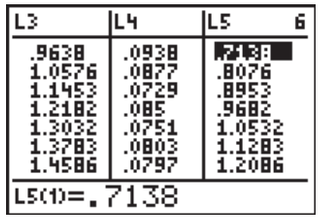


Figure 25

3. Press 2nd Y= to access the [STAT PLOT] menu. Turn on **Plot3** and define the **Xlist:** to be **HSEQ** and the **Ylist:** to be **L5. See Figure 26.**

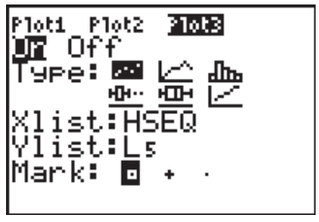


Figure 26

4. Go to the Y= window and turn off **Y2** as this was the equation for **Plot2**. Also, position the cursor on **Plot2** at the top of the screen and press ENTER to turn it off. **See Figure 27.**

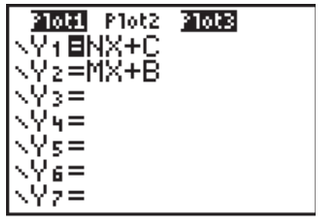


Figure 27

5. Press GRAPH to see the new data plotted along with the data from Activity 8 and its regression equation. **See Figure 28.**

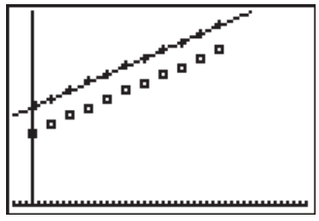


Figure 28

6. Rather than go through the long procedure to find the regression equation for this new data, let the calculator do it for you. Press STAT, arrow over to **CALC**, choose **4:LinReg(ax+b),** and then press ENTER. **See Figure 29.**

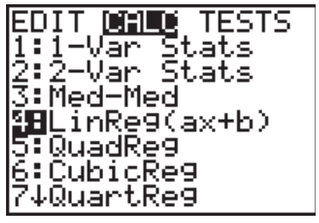


Figure 29

7. To enter a named list, press [2nd] [STAT] to access the [LIST] menu. Scroll through the choices until you highlight the list you want and then press [ENTER]. Enter **HSEQ**, **L5** and **Y3** as the arguments and press [ENTER] to execute the command. **See Figure 30.**

Figure 30

8. The regression equation is shown on the home screen and is also pasted in the [Y=] window. Draw the students' attention to the value of the slope before you leave this screen. **See Figure 31.**

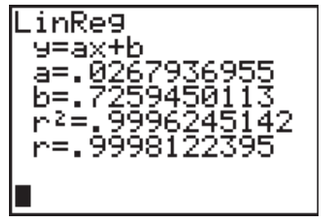


Figure 31

9. Press [GRAPH] to see both scatter plots and their regression equations. It seems clear that the two lines are parallel. These graphs model a transformation and give the students a concrete example to help them understand transformations. **See Figure 32.**

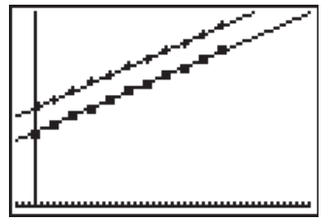


Figure 32

10. Next, we have an opportunity to use some other features of the calculator. To examine the slopes of the two lines, recall both slopes and have them displayed on the home screen. To find the a value from the regression equation that the calculator found, press [VARS] and select **5:Statistics. See Figure 33.**

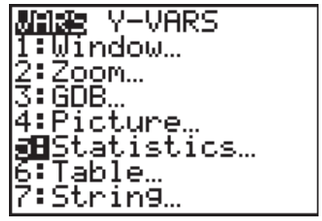


Figure 33

11. Arrow over to **EQ** and select **2:a**. **See Figure 34.**

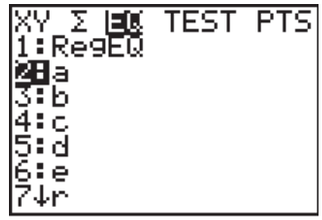


Figure 34

12. Press [ENTER] again when **a** appears on the home screen to see the value of **a**. Next, recall the **N**-value by pressing [ALPHA] **N** and then [ENTER]. Discuss why the values are not exactly the same. Find the difference between the two slopes by typing **N–a**. The keystrokes for this are [ALPHA] **N** – [VARS] and select **5:Statistics**, arrow over to **EQ** and select **2:a**. The answer displayed in the example is the calculator's form of scientific notation and stands for 3.25 x 10^{-4} which equals 0.000325 of a foot. **See Figure 35.**

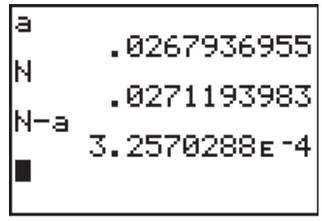


Figure 35

13. The average student knows that 0.5 feet = 6 inches and that it is found by multiplying 0.5 and 12. Use the same procedure to multiply the difference in the two slopes by 12. The answer demonstrates that the difference in the slopes is less than 4 one thousandths of an inch. Encourage them to consider this to be so close to zero that for such an activity as we have performed, it is a very small amount of error. **See Figure 36.**

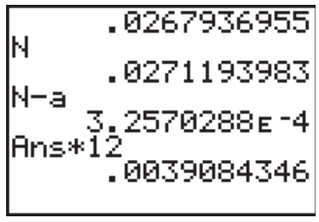


Figure 36

14. Use a similar procedure to explore the difference in the **Y**-intercepts. Recall that when we measured the cup it was 3 inches or 0.25 feet. **See Figure 37.**

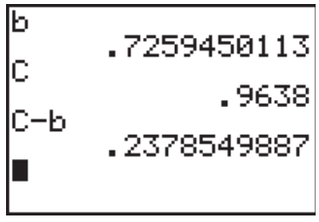


Figure 37

WORKSHEET ANSWERS

1. Answers will vary.
L3 in the chart is the total distance (answer to #1) minus the values in **L6.**
L5 in the chart is **L3** – **ht of the cup** (in feet).

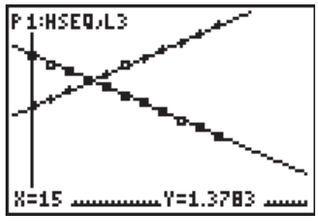
2. Number of candies

3. Distance in feet from the bottom of the cup to the stick.

4. Answers will vary.

5. Answers will vary.

6.



7. Steepness (slopes) seems to be the same degree, but is in opposite directions.

8. Positive slope means the distance increases as the number of candies increases and negative means the distance decreases as the number of candies increases.

9. The two **Y**-values will always equal the total distance from the stick to the floor.

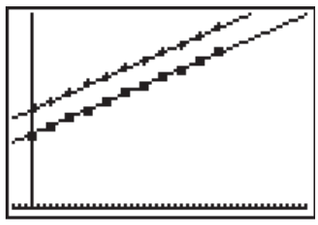
10. Answers will vary, but the **Y**-values on both lines will be the equal to the other and equal to one half the total distance.

11. Answers will vary.

12. Total distance from the stick to the floor.

13. The point of intersection is when the bottom of the cup is the same distance to the floor as it is from the stick.

EXTENSION

1. 

2. Distance that the top and the bottom of the cup were from the stick when there were no candies in the cup.
3. Answers will vary.
4. Height of the cup
5. Answers will vary.
6. Same slope as **Plot1**.
7. They are parallel.
8. Answers will vary.
9. Answers will vary.
10. Change in **Y**s divided by change in **X**s . . . rise/run
11. Change in distance per change in number of candies.
12. slope; **Y**-intercepts
13. positive
14. increasing; decreasing

ACTIVITY 8

Name: _____

Hooke's Law: The Rest of the Story

Math Objectives:

- Graph scatter plots
- Analyze and graph linear functions
- Calculate and apply slope
- Discuss positive versus negative slopes

Materials: (Per Group)

- TI-83/TI-84 Plus Family
- Data from Activity 7: Hooke's Law

OVERVIEW

In this activity, you will use the data from Activity 7 to create more lists and scatter plots and you will further explore the properties of a linear function to develop a model for motion. Finally, you will interpret the values used in your model. Your teacher will outline the procedure for you. *Round all decimals to the nearest hundredth.*

1. What was the total distance from the stick to the floor/CBR 2? _____

Data from Activity 7 Hooke's Law		**Lists created by formula using Activity 7 Data**	
Number of candies, L1 or HSEQ	**Distance from the bottom of the cup to the floor, L6 or HOOKS**	**Distance from the meter stick to the bottom of cup, L3**	**Distance from the meter stick to the top of the cup, L5**
0			
3			
6			
9			
12			
15			
18			
21			
24			
27			
30			

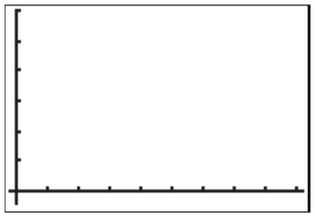
2. What physical property is represented along the **X**-axis of **Plot1**? _____

3. What physical property is represented along the **Y**-axis of **Plot1**? _____

4. What is the regression equation you found to fit the data for the scatter plot of the distance from the bottom of the cup to the stick vs. **HSEQ** (**Plot1**)? _____

5. What is the regression equation you found to fit the data for the scatter plot of the distance from the bottom of the cup to the floor vs. **HSEQ** (**Plot2**)? _____

6. On the screen to the right, sketch **Plot1** (**L3** vs. **HSEQ**) and **Plot2** (**HOOKS** vs. **HSEQ**).



7. When you see both plots on the same axis, how are the slopes similar?
_____

8. If both slopes deal with a change in distance dependent upon a change in the number of candies, why is one positive and the other negative?
_____

_____

9. For 6 candies, add the **Y**-values from the two plots. Repeat this for other numbers of candies. What do you get? _____ Explain why._____

_____

10. What is the point of intersection? **X** = _____ **Y** = _____

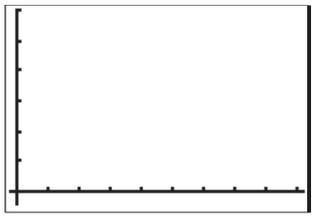
11. Multiply the **Y**-value of the intersection by 2. What do you get? _____

12. What other value is the same as this? _____

13. Explain what this means in your own words. _____

_____

EXTENSION



1. Draw a sketch of **Plot1** and **Plot3** on the axis at the right. **(Plot1** is **L3** vs. **HSEQ** and **Plot3** is **L5** vs. **HSEQ.)**
2. What do the **Y**-intercepts represent in each equation? _____
3. What is the difference in the **Y**-intercepts. _____
4. What does this difference represent? _____
5. What is the regression equation for the scatter plot of the distance the top of the cup is from the stick (**L5**) vs. **HSEQ**? _____
6. What do you notice about this equation and the one you found for the data in **Plot1**?
7. Physically, what is the relationship between the two lines? _____
8. For every 3 candies added, the distance the cup moved was approximately _____ ft?
9. How many inches is that? _____
10. What is the mathematical definition of slope? _____
11. Describe, in your own words, the meaning of slope in relation to this activity. _____
12. In general, if two lines are parallel, they will have the same _____ but different _____.
13. If a line slants up as you move from left to right, will it have a positive or negative slope? _____
14. The **X**-values in this activity stand for the number of candies and the **Y**-values stand for distances. In general, if the slope is positive it means the distance is _____ as the number of candies is increasing. If the slope is negative, the distance is _____ as the number of candies is increasing.