# Graphs of Continuous Distributions

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**Student Activity**

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TI-Nspire™     Activity     Student     1 hour

## Introduction

This is the second activity in the *Probability and Statistics with Pizzazz* series.
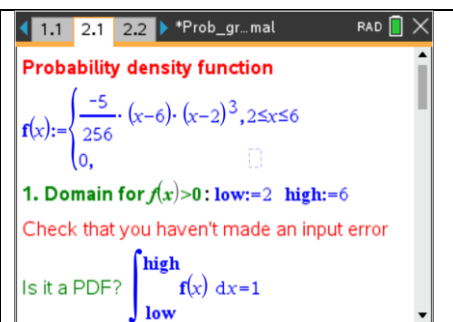The first activity in the series focuses on the binomial distribution and its graphs.
This activity focuses on graphs of probability density functions for continuous random variables, including the normal and standard normal distributions.

## Exploration 2. Graphs of the distribution of continuous random variables

Notes page and graph to calculate and analyse probabilities, mean, variance for a PDF

Open the TI-Nspire document *Prob_graphs_continuous_normal* and navigate to page 2.1. This is a Notes page that has been set up to carry out relevant calculations on the probability density function (PDF), $f$. The rule for $f$ can be edited and the template used for calculations in class and in test and exam settings.     Consider the continuous random variable, $X$ , with PDF

$$f(x) = \begin{cases} \frac{-5}{256} \cdot (x-6) \cdot (x-2)^3, & 2 \le x \le 6 \\ 0 & , \text{ elsewhere} \end{cases}$$

*Probability density function*

$f(x):= \begin{cases} \frac{-5}{256} \cdot (x-6) \cdot (x-2)^3, 2 \le x \le 6 \\ 0, \end{cases}$

**1. Domain for $f(x) > 0$: low:=2  high:=6**
Check that you haven't made an input error

Is it a PDF? $\int_{low}^{high} f(x)\,dx = 1$

**To evaluate a Maths Box, click on it and press <enter>. The output should appear in green.**
Click on the top Maths Box and press <enter>. The output 'Done' shows $f$ has been defined.

## Question 1

Determine whether $f$ a valid probability density function.
Under item 1 on page 2.1, input the domain for $f(x) = 0$ and evaluate the Maths Box next to the 'Is it a PDF'.
a.  How does the output of item 1 show that $f$ is valid?
b.  Navigate to page 2.2, which shows the graph of $f$. Adjust the sliders to $a = 2$ and $b = 6$. How does this interactive graph validate that the function is a valid PDF?
c.  In item 1 (page 2.1), change the 'low' value to 3, so that the domain for $f(x) = 0$ is $3 \le x \le 6$. How does the new output show that this is not a valid PDF?
At the top of page 2.1, restore the domain to low: $= 2$ and high: $= 6$.

## Question 2

a.  Determine the following probabilities using item 2 on page 2.1, correct to four decimal places.
   i.  $\Pr(2.4 < X < 4.5)$   ii. $\Pr(X > 3.6)$   iii. $\Pr(X < 5.0)$

b.  On page 2.2, what values of $a$ and $b$ could you set on the sliders to determine the above probabilities from the graph? Verify your answers to Q.2 a. using the graph.

$x = \mu$     a =1.5     relation plot rel3     b =5.4
$\Pr(a < X < b)$     area=0.8352     $(4.67, 0)$
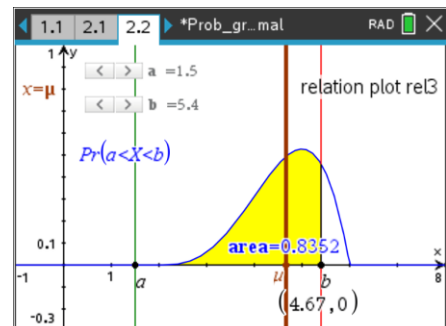
## Question 3

a.  Use item 3 on page 2.1 to determine the mean, $\mu$, of the distribution, correct to four decimal places. (You will need to evaluate all Maths Boxes in item 3 for this to work correctly).
b.  On page 2.2, the brown vertical line has equation $x = \mu$. How does this line allow you to determine the value of the mean graphically?

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## Question 4

a. Use item 4 on page 2.1 to determine the variance and standard deviation, correct to four decimal places.

b. On page 2.2, edit the slider values, correct to one decimal place, to $a = \mu - \sigma$ and $b = \mu + \sigma$, where $\sigma$ is the value of the standard deviation calculated above.

   i.    Hence use the graph to determine $\Pr(\mu - \sigma < X < \mu + \sigma)$, correct to 2 decimal places.

   ii.   Use the same method to determine $\Pr(\mu - 2\sigma < X < \mu + 2\sigma)$, correct to two decimal places.

## Question 5

a. Use item 5 on page 2.1 to determine the value of $d$ such that $\Pr(d < X) = 0.5$, correct to four decimal places.

b. On page 2.2, edit the slider values appropriately and verify the previous answer graphically. State the slider values used for $a$ and $b$. (Note. Slider '$b$' can be edited to 4 decimal places.)

c. Compare the values of $d$ and $\mu$ on the graph, and comment on any interesting features.

## Exploration 3. Graphs of the Normal distribution

There are many practical situations that can be modelled by a normally distributed random variable, including the height of adults in a population, birth weight of babies, and scores in IQ and aptitude tests.



If $X$ is a random variable with a normal distribution, then it has probability density function (PDF) of the form:

$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}, x \in R$, where the parameters $\mu$ and $\sigma$ are the mean and standard deviation, respectively. The TI-Nspire functions 'normPdf(XVal [,μ [,σ]])' and 'normCdf(lowBound, upBound [,μ [,σ]])' simplify the use of the normal density function for graphing and calculating probabilities. This is illustrated in the screenshots on the right.
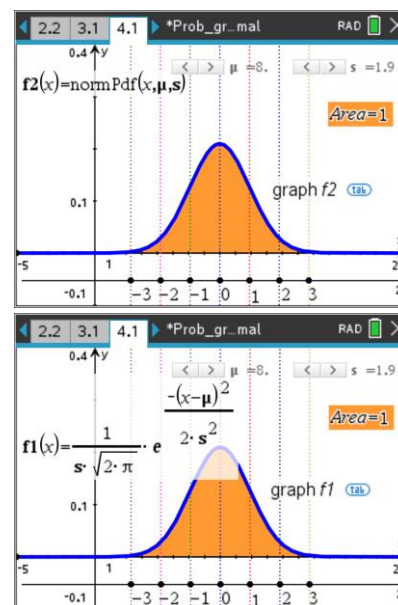


### The effect of changing $\mu$ and $\sigma$

Navigate to page 4.1 of the TI-Nspire document *Prob_graphs_continuous_normal*. The Graphs page contains a graph of the normal density function, with sliders controlling the value of $\mu$ and $\sigma$. **Note. The slider variable $s$ controls the standard deviation, $\sigma$ (TI-Nspire doesn't allow the symbol $\sigma$ as a slider variable).**

The vertical lines have equations such as $x = \mu$, $x = \mu + \sigma$, $x = \mu - 2\sigma$ etc. Move the cursor to a line to see its equation.
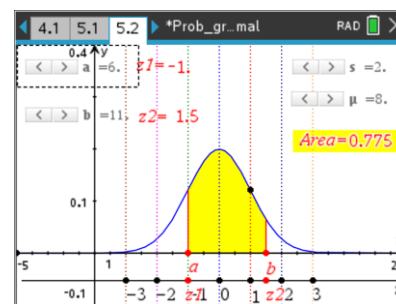
## Question 6

a. Describe the shape and other key features of the graph of the normal distribution function.

b. Use the slider to systematically change the value of the mean, $\mu$, while keeping the standard deviation constant. Describe the effect of changing $\mu$ on the graph of the function.

c. Use the slider with the variable, $s$, to systematically change the value of the standard deviation, while keeping the mean constant. Describe the effect of changing the standard deviation on the graph of the function.

d. The area of the orange region under the curve is calculated as the integral over the interval $(-\infty, \infty)$. What effect, if any, does changing the values of the mean and standard deviation have on the area under the graph? Explain the reason why this behaviour is a necessary condition for the function.

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## The Standard Normal distribution

It is sometimes useful to convert a normal distribution into the **standard normal distribution** to make comparisons on scores from different distributions with different means and standard deviations. The new standardised scores are often called z-scores.

Navigate to page 5.2 of the TI-Nspire document. The scale under the x-axis, labelled $z$, shows the standaised z-scores for the distribution.

## Question 7

a. Use the slider to change the mean of the distribution. What happens to the value of the mean on the z-scale as the slider values change? What z-score is always given to the mean?

b. Use the slider to change the standard deviation of the distribution. Note the dotted vertical lines at $x = \mu \pm \sigma$, $x = \mu \pm 2\sigma$ and $x = \mu \pm 3\sigma$. As the value of the standard deviation changes, what happens to the values at which these vertical lines intersect the z-scale? What z-score is always given to the standard deviation?

## Question 8

Imagine that a student looking at the screenshot of page 5.2 above made the following observations about the transformation which maps a point, $P(x, y)$, on the graph of the normal distribution (with mean $\mu$ and standard deviation $\sigma$) to a point, $P'(z, y')$, on the graph of the 'z-scaled' standard normal.

(1) $(x = \mu = 8) \rightarrow (z = 0)$

(2) $(x = \mu + \sigma = 8 + 2 = 10) \rightarrow \left(z = 1 = \frac{10-8}{2}\right)$

(3) Generalise to $x \rightarrow \left(z = \frac{x-\mu}{\sigma}\right)$

(4) The area under the graphs of the normal and of the standard normal must equal 1. Therefore, if the normal graph is dilated by a factor of $\frac{1}{2}$ parallel to the x-axis, then it must be dilated by a factor of 2 parallel to the y-axis.

a. On page 5.2, change the values of $\mu$ and $\sigma$ and comment on whether the four observations above are valid for all the values chosen.

b. Generalise, in terms of the general parameters $\mu$ and $\sigma$, the transformation

$P(x, y) \rightarrow P'(_____, \_\_\_\_\_)$

## Question 9

On page 5.2, adjust the slider values to $\mu = 10$ and $\sigma = 2$. Adjust the slider values for $a$ and $b$ to determine (graphically) the following probabilities, correct to 3 decimal places.

a. $\Pr(\mu - \sigma < X < \mu + \sigma) = \Pr(\_\_\_\_\_ < Z < \_\_\_\_\_) = _____$ (area under graph from $a$ to $b$)

b. $\Pr(\mu - 2\sigma < X < \mu + 2\sigma) = \Pr(\_\_\_\_\_ < Z < \_\_\_\_\_) = _____$

c. $\Pr(\mu - 3\sigma < X < \mu + 3\sigma) = \Pr(\_\_\_\_\_ < Z < \_\_\_\_\_) = _____$

d. Comment on what a negative z value represents.

e. Choose different values $\mu$ and $\sigma$ and repeat the above calculations. Comment on the results.

## Question 10

On page 5.2, adjust the slider values to $\mu = 12$ and $\sigma = 3$. Adjust the slider values for $a$ and $b$ to determine (graphically) the following probabilities, correct to 3 decimal places.

a. $\Pr(8 < X < 12) = \Pr(\_\_\_\_\_ < Z < \_\_\_\_\_) = _____$ (area under graph from $a$ to $b$)

b. $\Pr(6 < X < 17) = \Pr(\_\_\_\_\_ < Z < \_\_\_\_\_) = _____$

c. $\Pr(X < 14) = \Pr(Z < \_\_\_\_\_) = _____$

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