Math Objectives

- Students will determine the type of function modeled by the height of a capsule on the London Eye observation wheel.
- Students will translate observational information to use as the parameters of a cosine function.
- Students will determine the amplitude, angular frequency, period, and midline of a cosine function when given information in verbal or graphical form.
- Students will model the height of a capsule on the London Eye by writing an equation in the form $y = -A \cdot \cos(Bx) + D$.
- Students will model with mathematics (CCSS Mathematical Practice).

Vocabulary

- amplitude
- angular frequency
- midline
- parameters of a function
- period
- periodic function
- sinusoidal function

About the Lesson

- This lesson involves creating an appropriate equation to model the height of a capsule on the wheel.
- As a result, students will:
  - Use a slider to animate the function modeled by the height of a capsule on the London Eye observation wheel.
  - Discover the concepts of amplitude, angular frequency, period, and midline.
  - Determine the amplitude, angular frequency, period, and midline of the "observation wheel" function.

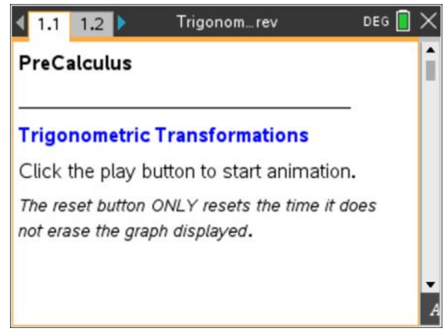
Related Lessons

- Prior to this lesson: Unit Circle

TI-Nspire Navigator™

- Use of Document Collection/Quick Poll/Class Capture will allow the teacher to assess student understanding during the lesson.
- Use Class Capture to examine results.

Discussion Points and Possible Answers



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a slider
- Click slider arrows to begin an animation
- Move between applications
- Show the function entry line
- Insert the equation of a function and graph it

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.

Lesson Materials:

Student Activity

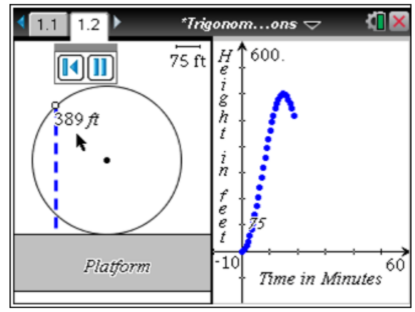
- Trigonometric_Transformations_Student.pdf
- Trigonometric_Transformations_Student.doc

TI-Nspire document

- Trigonometric_Transformations.tns

Tech Tip: Press esc to hide the entry line if students accidentally press tab.

Move to page 1.2.



1. On the screen, you see a model of the London Eye on the left side and a graph on the right. Click on the play button to start the animation. Click the button again to stop it. What type of function was created as a result of the animation?

   Sample Answers: A sinusoidal function. A cosine function. A periodic function. A cyclical function.

TI-Nspire Navigator Opportunity: *Class Capture* or *Live Presenter*
See Note 1 at the end of this lesson.

2. What does the changing measurement on the left screen represent as the capsule (represented by the open circle) moves around the observation wheel?

   Teacher Tip: Students should recognize the shape of the graph. Some students might recognize the transformations, while others will not.

   Answer: The measurement shows the height of a capsule from the platform.

3. What are the units of the x- and y-axes on the right?

   Answer: The x-axis represents time in minutes. The y-axis represents height in feet.

4. a. What is the maximum height a capsule reaches from the platform?

   Answer: 450 feet

TI-Nspire Navigator Opportunity: *Class Capture* or *Live Presenter*
See Note 2 at the end of this lesson.

b. The horizontal line halfway between the maximum and minimum of the function is called the ***midline*** of the graph. What is the equation of the midline? Explain your reasoning.

Answer: The equation of the midline is $y = 225$. The maximum height is 450 feet and the minimum is 0 feet, resulting in a midpoint of 225.

5. The function $y = –A \cdot \cos(Bx) + D$ can be used to model the capsule's height above the platform at time x. This is a transformation of a basic cosine curve.

a. Use your knowledge of transformations to explain why there is a negative sign in front of the variable A.

Answer: The basic cosine function starts at its maximum. This function starts at its minimum. The negative represents the reflection about the midline.

b. The variable A represents the ***amplitude***, which is the vertical distance between the midline and the maximum or the minimum. What is the amplitude of the "observation wheel" function, and how did you find the value?

Answer: The amplitude is 225. This distance from the maximum of 450 to the value of the midline is 225.

c. Which variable of the equations represents the midline of the function? Explain your reasoning.

Answer: The midline is the variable D, which is 225 in this function. In a basic cosine curve, the maximum is 1, the minimum is –1, and the midline is $y = 0$. In this function, the maximum is 450, the minimum is 0, and the midline is 225. Vertical shifts are represented as an addition or subtraction from the basic function.

d. The ***period*** of a function is the time it takes to complete one cycle of a periodic function. What is the period of the "observation wheel" function, and how is it visible in the graph?

Answer: The period is 30. Looking at the graph, it takes 60 minutes to complete two cycles. Thus, it takes 30 minutes to complete one cycle of the periodic function.

6. What characteristic of the observation wheel does the amplitude represent? Explain your reasoning.

   Answer: The amplitude represents the radius of the observation wheel.

7. The variable *B* represents angular frequency. ***Angular frequency*** is the measure of the arc (in radians) traveled by the capsule divided by the time traveled (in minutes).

   a. What is the measure of the arc traveled by the capsule in one complete revolution?

      Answer: The measure of the arc is 2π.

      Teacher Tip: You might have to remind students that the measure of the arc is the measure of the central angle, and the length of the arc is the distance traveled. Frequency in this example is angular velocity.

   b. How long does it take for a capsule to complete one revolution?

      Answer: It takes 30 minutes.

   c. What is the frequency for the "observation wheel" function?

      Answer: Frequency is $\frac{2\pi}{30} = \frac{\pi}{15}$.

8. Using $y = -A \cdot \cos(Bx) + D$ and the variable information found in Question 5, write the equation representing the height of a London Eye capsule at time x. Verify your answer by graphing the function.

   Answer: The equation is $y = -225 \cdot \cos\left(\frac{\pi x}{15}\right) + 225$.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

9. Imagine the boarding platform for the observation wheel stands 10 feet above the ground. If your function takes this height into consideration, what parameters of the equation would change? What parameters would stay the same?

   Answer: The new equation would be $y = -225 \cdot \cos\left(\frac{\pi x}{15}\right) + 235$.

   The only parameter that changes is *D*, the vertical shift. The amplitude and frequency stay the same because they are based on the observation wheel, not where it exists in space.

Extension:

How can you model the London Eye using a sine function?

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The type of function modeled by the height of a capsule on an observation wheel.
- Parameters of a cosine function.
- How to determine the amplitude, angular frequency, period, and midline of a cosine function when given information in verbal or graphical form.
- How to use parameters to write an equation in the form $y = -A \cdot \cos(Bx) + D$.

TI-Nspire Navigator

Note 1

Question 1, *Class Capture* and *Live Presenter*

As students begin this activity, use Class Capture to be assured that each student is able to play the animation. You can choose one student to display their work on this and future problems and discuss the results.

Note 2

Question 3, 4, *Class Capture*

You can choose to use Class Capture to share the work for these problems.

Note 3

Whole Document, *Quick Poll*

Quick Poll can be used throughout the lesson to assess student understanding.