Frank's Farm

Answers

7 8 9 10 11 12

TI-Nspire Investigation Student 20 min

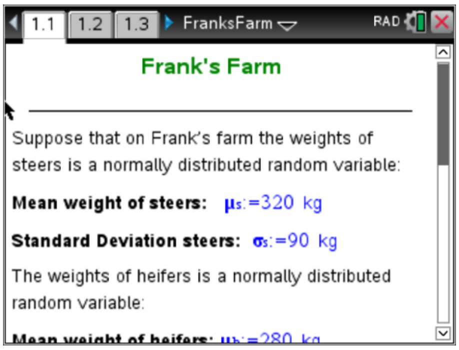
Steers & Heifers

Suppose that on Frank's farm the weight, X_i, of a randomly selected steer is a normally distributed random variable with mean 320 kg and standard deviation 90 kg. The weight, Y_i, of a randomly selected heifer is a normally distributed random variable with mean 280 kg and standard deviation 80 kg.

Problem Statement 1

Find the probability that for a random sample of 4 steers and 2 heifers the total weight is less than 2000kg.

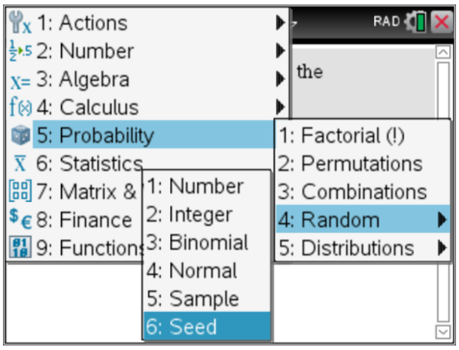
Simulating the Problem



Open the TI-Nspire file: **Franks Farm**

Page 1.1 contains a set of instructions. These instructions include setting the variables in the problem. Make sure the variables are all set before starting the simulation.

Note: When modifying a variable press [**Enter**] to ensure the modification has been accepted.



Navigate to page 1.2.

Frank's farm problem can be simulated from this application. Before starting however it is important to seed the random variable generator.

Menu: **Probability** > **Random** > **Seed**

Enter a four digit number such as the last four digits of your phone number. This will ensure your results are different than everyone else. (Unless of course you share the same phone!)

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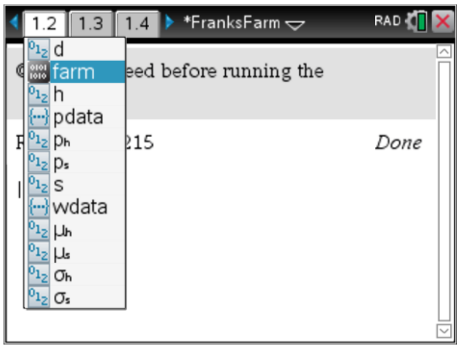


To run the program, press the [**VAR**] key and select "**farm**".

Notice that all the variables for the problem have been automatically included from the introductory page.

farm() will appear on the screen, press [**Enter**] to run the program and enter the number of simulations.

The program produces the following:

- The proportion with a combined weight greater than or equal to the amount specified (w)
- The proportion with a combined value greater than the amount specified (d).
- wdata = Total weight of each sample
- pdata = Price of each sample

Question: 1.

Run a simulation of 100 trials. Record the following:

i. Proportion of samples that exceed the combined weight and also below the combined weight.
Answers will vary (simulation) – Proportions provided by the simulation typically vary between 0.14 and 0.32.

Comment: Too much variation to provide an accurate estimate of the answer.

ii. Mean weight of cattle in the sample.
Typical means for the 100 trials vary around 1850kg.

Comment: Unlike the results for sample proportions, the mean is very consistent, as 'expected'.

Question: 2.

Run several more simulations of 100 trials and record the mean, proportion of samples that exceed the required weight and therefore the proportion that do not meet the requirement. Now run several simulations of 400 trials and record the results. Compare the 'consistency' of the two sets of data.

Note: *The program will take a few moments to generate the 1000's of random numbers required for this simulation.*

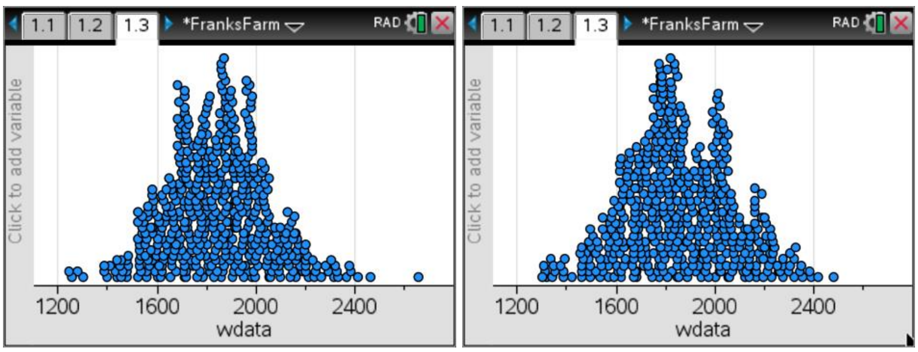
Answers will vary for the additional simulations; however students should notice that whilst the proportions vary considerably from sample to sample, the mean is very consistent. Simulations for 400 trials increases the consistency of the proportions and the mean. Typical sample proportions for 400 trials consistently produce 0.22 exceeding 2000kg and therefore 0.78 below 2000kg.

Question: 3.

Run a simulation of 500 trials and graph the 'weight' results (wdata) in a scatter plot.

i. From the scatter plot estimate the mean weight of the combined steers and heifers.

Samples:



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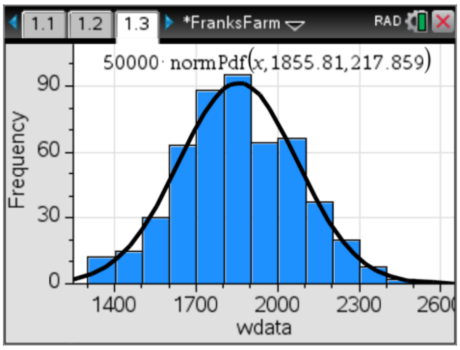
ii. Describe the general shape of the combined weight data.
Data is approximately normally distributed.

Comments: Changing the graph type to a histogram allows for a normal probability function to be drawn over the data.



Question: 4.

Find the probability that for a random sample of 4 steers and 2 heifers, the combined weight is less than 2000kg. Write your answer correct to four decimal places showing all working out.

Let V represent the distribution of 4 steers and 2 heifers.

$$E(V) = E(X) + E(X) + E(X) + E(X) + E(Y) + E(Y)$$
$$\text{or } = E(X_1 + X_2 + X_3 + X_4) + E(Y_1 + Y_2)$$
$$= 4E(X) + 2E(Y)$$
$$= 4 \times 320 + 2 \times 280$$
$$= 1840$$
$$Var(V) = Var(X) + Var(X) + Var(X) + Var(X) + Var(Y) + Var(Y)$$
$$\text{or } = Var(X_1 + X_2 + X_3 + X_4) + Var(Y_1 + Y_2)$$
$$= 4Var(X) + 2Var(Y)$$
$$= 4 \times 90^2 + 2 \times 80^2$$
$$= 45200$$

$$V \simeq N(\mu, \sigma) = N\left(1840, \sqrt{45200}\right)$$

$$\Pr(V < 2000) = 0.7741$$

Comments: Each heifer and steer represents an additional random sample not multiple versions of the same therefore the mean of each and variance of each are simply added. Note that the simulation refers to *exceeding* the weight so the proportion less than the required weight must be subtracted from 1. The simulated results are very close to theoretical. If students incorrectly apply the formula: $Var(V) = a^2Var(X) + b^2Var(Y)$ their probability (0.657) will be substantially different from the simulation, particularly if they check simulation results with other students.
An opportunity exists here to discuss type I and type II errors. Could the sample results come from a population with mean 1840 and standard deviation: $40\sqrt{97}$? (Incorrect transformation)

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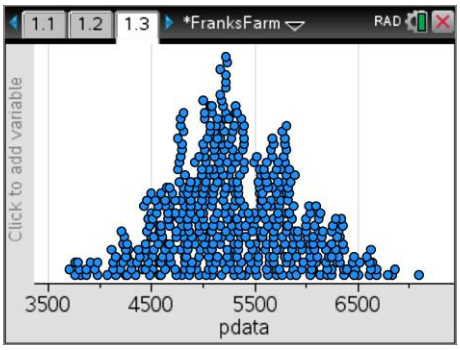
Problem Statement 2

Suppose that steers are valued at \$3.00/kg live-weight and heifers at \$2.50/kg live-weight. Find the probability that for a random sample of 4 steers and 2 heifers, the combined value is greater than \$5200.

Question: 5.

The simulation data from Question 3 is still stored in the calculator. Navigate back to the scatter plot.

i. Change the x variable in the scatter plot to the combined value (pdata) and estimate the mean.

Sample: 

ii. Describe the general shape of the combined value data.

Answers: Approximately normally distributed.

Question: 6.

Find the probability that for a random sample of 4 steers and 2 heifers, the combined value is greater than \$5200. Write your answer correct to four decimal places. Show all working out.

Let P represent the price distribution for 4 steers and 2 heifers.

$$\begin{aligned} E(P) &= E(aX+bY) \\ &= aE(X)+bE(Y) \\ &= aE\left(X_1+X_2+X_3+X_4\right)+bE\left(Y_1+Y_2\right) \\ &= 3\times(4\times 320)+2.5\times(2\times 280) \\ &= 5240 \end{aligned}$$

$$\begin{aligned} Var(P) &= Var(aX+bY) \\ &= a^2Var(X)+b^2Var(Y) \\ &= a^2Var\left(X_1+X_2+X_3+X_4\right)+b^2Var\left(Y_1+Y_2\right) \\ &= 3^2\times(4\times 90^2)+2.5^2\times(2\times 80^2) \\ &= 371600 \end{aligned}$$

$$P \simeq N\left(\mu,\sigma\right) = N\left(5240, \sqrt{371600}\right)$$

$$\Pr(P>5200) = 0.5262$$

Comments: The simulation returns very similar and consistent proportions to the theoretical probability.

Teacher Notes: The original question can be modified as the notes page contains all the parameters in the question. The program can also be modified to computer other proportions or modify the proportions already included.

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