

**Operating on Matrices****ID: 11358****Time Required***20 minutes***Activity Overview**

*In this activity, students will learn how to add, subtract, and multiply matrices. Students will also learn how to find the determinant and inverse of a matrix. A set of practice problems will reinforce their skills.*

**Topic: Matrices**

- *Operations*
- *Determinant*
- *Inverse*

**Teacher Preparation and Notes**

- *This activity was made to be a learning tool for how to work with matrices. The activity can be a teacher demonstration of the skills, followed by independent practice work by the students, or the students can work independently on the skills in class or at home.*
- *The Practice Problems section can be assigned as classwork or homework.*
- *To download the student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “11358” in the keyword search box.*

**Associated Materials**

- *OperatingOnMatrices\_Student.doc*

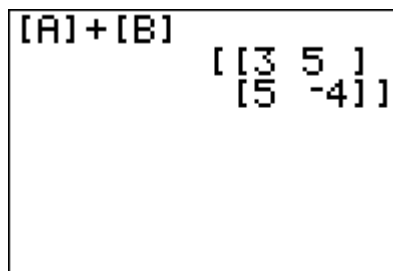
**Suggested Related Activities**

*To download any activity listed, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter the number in the keyword search box.*

- *All Systems Go (TI-84 Plus family) — 10127*
- *Finding Atomic Weight Using Matrices (TI-Nspire technology) — 9709*
- *Properties of Matrices (TI-Nspire technology) — 8986*
- *Cramer’s Rule (TI-Nspire technology) — 8793*

**Part 1 – Adding and Subtracting Matrices**

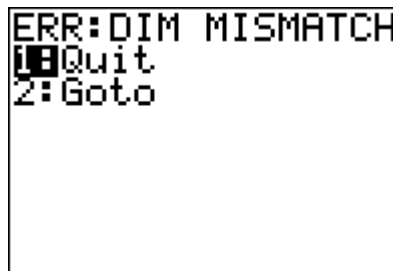
Students will use their handhelds to solve addition and subtraction problems. By solving the problems, students should be able to find out a rule for adding and subtracting two matrices. Students should see that the matrices need to have the same dimensions. Elements in corresponding locations should be added or subtracted.



$$[A]+[B]$$

$$\begin{bmatrix} 3 & 5 \\ 5 & -4 \end{bmatrix}$$

The calculator screen to the right displays the error message when the dimensions of the matrices being added or subtracted are not the same.

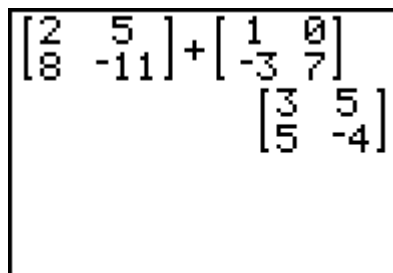


ERR: DIM MISMATCH  
1: Quit  
2: Goto

Note: Make sure students use the negative sign and not the minus sign when entering negative values into the matrix.

**If using Mathprint™ OS:**

Students can enter the matrix directly on the Home screen. They need to press  $\boxed{\text{ALPHA}}$   $\boxed{\text{F3}}$  and select the dimensions of the row and column of the matrix. Then they can type the entries and use the arrow keys to move from one to the other.



$$\begin{bmatrix} 2 & 5 \\ 8 & -11 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -3 & 7 \end{bmatrix}$$

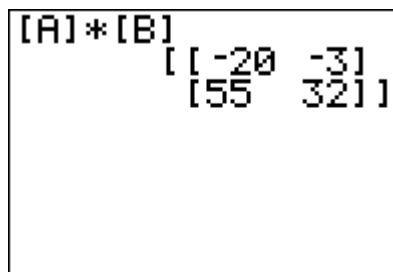
$$\begin{bmatrix} 3 & 5 \\ 5 & -4 \end{bmatrix}$$

Discussion Questions:

- How does the handheld add or subtract matrices?
- What are the criteria for when two matrices can be added or subtracted?

**Part 2 – Multiplying Matrices**

The goal of this section is for students to understand when matrices can and cannot be multiplied and what the dimensions of the resulting matrix are. Students should observe the patterns when multiplying two matrices. They should see that, in order to multiply two matrices, the number of columns in the first matrix needs to match the number of rows in the second matrix. The resulting matrix will have the same number of rows as the first matrix and number of columns as the second matrix.



$$[A]*[B]$$

$$\begin{bmatrix} -20 & -3 \\ 55 & 32 \end{bmatrix}$$

The calculator screen to the right is the error message displayed when multiplying two matrices where the number of columns in the first matrix is not the same as the number of rows in the second matrix.

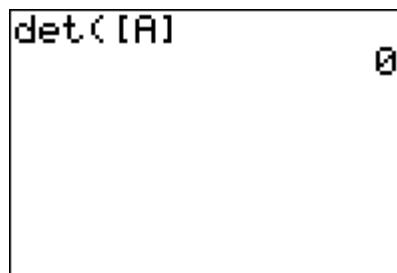


Discussion Questions:

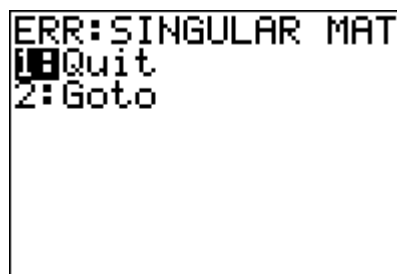
- Why does the handheld not multiply some matrices?
- What are the criteria for when two matrices can be multiplied together?

**Part 3 – Inverses and Determinants of Matrices**

In this section, students will find out when a matrix has an inverse and when the determinant can be found. Students should find out that only square matrices have an inverse and a determinant.



To the right is what the Handheld displays when trying to find the inverse of a matrix whose determinant is 0.



Discussion Questions:

- When does a matrix have an inverse or determinant?
- Can you find the determinant of a  $2 \times 2$  matrix without the handheld? If so, create a rule for finding the determinant of a  $2 \times 2$  matrix.

**Student Solutions**

1. a.  $\begin{bmatrix} 3 & 5 \\ 5 & -4 \end{bmatrix}$

b.  $\begin{bmatrix} -2 & -6 \\ 15 & -3 \end{bmatrix}$

c. does not exist

d. does not exist

2. Matrices can be added or subtracted when their dimensions are the same.

3. The error message only happens when the dimensions of the matrices do not match. The error message explains that the dimensions of the matrices do not match.
4. To add or subtract matrices, elements in corresponding locations should be added or subtracted. The matrices also need to have the same dimensions.

5. a.  $\begin{bmatrix} -20 & -3 \\ 55 & 32 \end{bmatrix}$

b.  $[-37]$

c. does not exist

d.  $\begin{bmatrix} 55 & 14 & 60 \\ -68 & -43 & -69 \\ -11 & -35 & -55 \end{bmatrix}$

e. does not exist

6. 2 by 2 • 2 by 2, 1 by 2 • 2 by 1, 3 by 3 • 3 by 3; the 'inside' dimension numbers are the same.
7. The number of columns in the result is the same as the number of columns in the first matrix. The number of rows in the result is the same as the number of rows in the second matrix.
8. If an  $a \times b$  matrix is multiplied by a  $c \times d$  matrix, then  $b = c$  for the result to exist.
9. a. inverse does not exist, determinant = 0

b. inverse =  $\begin{bmatrix} \frac{5}{13} & -\frac{8}{13} & -\frac{1}{26} \\ \frac{1}{13} & \frac{1}{13} & \frac{5}{26} \\ \frac{2}{13} & \frac{2}{13} & -\frac{3}{26} \end{bmatrix}$ , determinant = -26

c. inverse =  $\begin{bmatrix} \frac{1}{3} & -\frac{2}{9} \\ \frac{1}{3} & \frac{1}{9} \end{bmatrix}$ , determinant = 9

d. inverse and determinant do not exist

e. inverse and determinant do not exist

10. The matrices that have a determinant are square matrices. The matrices that have an inverse are square and the determinant is not equal to zero.
11. If the determinant exists and is not equal to zero, the inverse exists. Also, it appears that the elements in the inverse matrix were divided by the determinant.
12. Yes; the determinant must exist and be non-zero for an inverse to exist.

**Practice Problem Solutions**

1.  $\begin{bmatrix} 27 & 24 \\ -11 & 13 \end{bmatrix}$

2.  $\begin{bmatrix} 1.7 & -0.3 \\ 2.4 & 2.6 \end{bmatrix}$

3.  $\begin{bmatrix} -\frac{3}{4} & \frac{33}{4} \\ -2 & -\frac{9}{35} \end{bmatrix}$

4.  $\begin{bmatrix} 8.6 & 18 \\ 5 & 31 \end{bmatrix}$

5.  $\begin{bmatrix} -14 & 2 \\ 73 & -12 \end{bmatrix}$

6.  $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$

7. does not exist

8.  $\begin{bmatrix} 18 & -14 & 6 \\ -\frac{9}{8} & \frac{7}{8} & -\frac{3}{8} \end{bmatrix}$

9.  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

10.  $\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$

11. does not exist

12. -3