You Can't Get There From Here

Name ________________________

Class ________________________

Problem 1 – The Basics

Press [WINDOW] and match the screen settings to the right.

Graph the function $f(x)=\dfrac{1}{x^2-9}$ and answer the questions below.

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WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-5
 Ymax=5
 Yscl=1
 Xres=1
```

1. Is the function $f(x)=\dfrac{1}{x^2-9}$ defined over all values of *x*?

Press [2nd] [TABLE] to view the table of values.

2. How does the table of values indicate that the function is undefined for a certain value of *x*?

3. What *x*-values have no corresponding *y*-value(s)?

4. Factor the denominator of $f(x)=\dfrac{1}{x^2-9}$.

5. How does the factored form of the denominator relate to the "skipped" *x*-values for which there are no resulting function values found on the graph?

6. When the "skipped" *x*-values are substituted into the function, what happens to the denominator? What effect does this have on the function at these *x*-values?

These *x*-values that result in the denominator becoming equal to zero (where the function is undefined) are referred to as **singularities**.

Look back over your graph. Notice that the function value approaches ±∞ to the left and right of the values for which the function is undefined. This means that the locations of these *x*-values are also locations of asymptotes, or "invisible lines" on the graph which the graph of the function approaches, getting closer and closer, but never reaches.

Problem 2 – Exploration

When the degree of the numerator is less than or equal to the degree of the denominator of a rational function, a **horizontal asymptote** may exist.

Once the rational function is simplified, then when the degree of the numerator equals the degree of the denominator, a horizontal asymptote exists at y = (ratio of leading coefficients of the numerator and denominator).

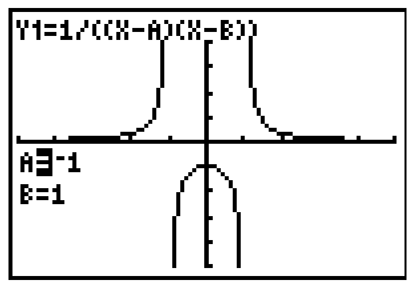
When the degree of the numerator is less than that of the denominator, a horizontal asymptote exists at $y = 0$.

7. Does the function from Problem 1 have a horizontal asymptote? If so, what is the equation of the asymptote?

Start the Transformation Graphing application by pressing APPS and selecting **Transfrm**.

Note: The Transfrm APP changes the value of Xres from 1 to 3. Press WINDOW and change Xres to 1.

Enter the function $f(x) = \dfrac{1}{(x-a)(x-b)}$



Explore what happens to the vertical asymptotes as values for a and b change.

8. Where is/are the vertical asymptote(s) for the function?

For the following questions, examine the graph of the function $f(x) = \dfrac{A \cdot x^C - 7}{B \cdot x^2 + x - 2}$ using your graphing calculator.

9. When only the value of c is changed, when would horizontal asymptotes not be present?

10. When the degree of the numerator and denominator are equal (once the rational function is simplified), what is the equation for the horizontal asymptote?

11. When the degree of the denominator is greater than the degree of the numerator, where does a horizontal asymptote exist?

12. Define/explain the following terms.

a. Singularity:

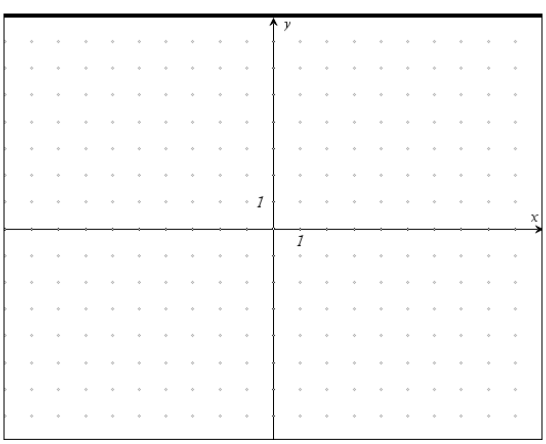
b. Asymptote:

Problem 3 – Practice

13. Factor the denominator of the function, $f(x) = \frac{5x - 7}{4x^2 - 8x - 12}$. At what values of x is the function undefined?

14. Do any horizontal asymptotes exist for this function? If so, where are they?

15. Sketch a graph of the function. Include asymptotes as dashed lines. Label asymptotes with their equations.



Problem 4 – The Next Level

16. Factor the denominator of the function, $f(x)=\dfrac{2x^2+2x-23}{x^2+x-12}$ and graph the function.

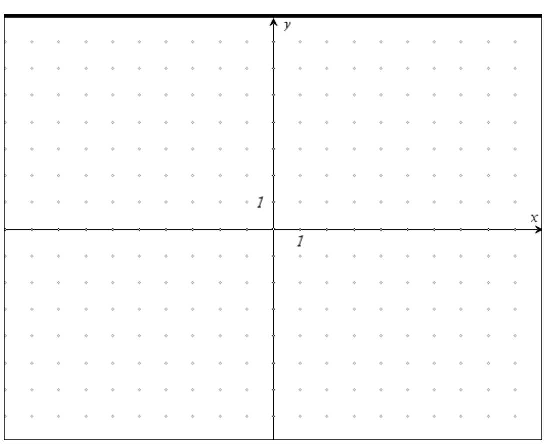
17. What are the singularities for this function?

18. Do any horizontal asymptotes exist for this function? If so, where are they?

The function $f(x)=\dfrac{2x^2+2x-23}{x^2+x-12}$ can be rewritten as $f(x)=\dfrac{1}{x^2+x-12}+2$.

19. How does the second representation of the function, $f(x)=\dfrac{1}{x^2+x-12}+2$, yield information about the horizontal asymptote of the function? As you consider the original function, does this agree with the information provided earlier regarding the ratio for the leading coefficients of the numerator and denominator when degrees are equal?

20. Sketch a graph of the function. Include asymptotes as dashed lines. Label asymptotes with their equations.



Additional Practice Problems

Identify the singularities, vertical asymptotes, and horizontal asymptotes for the given functions.

Function	Singularities	Vertical Asymptotes	Horizontal Asymptotes
21. $f(x)=\dfrac{1}{x^2-16}$			
22. $f(x)=\dfrac{-7x-11}{x^2+4x+4}$			
23. $f(x)=\dfrac{x^3}{x^2+2x-8}$			
24. $f(x)=\dfrac{2x^2+42}{x^2+2x-24}$			