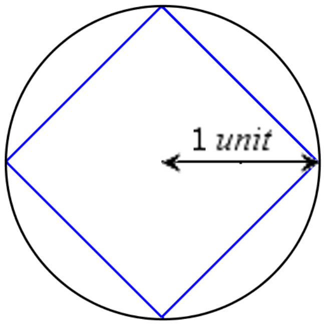
Take it to the limit?

Student Activity

7 8 9 10 **11** 12

TI-Nspire Investigation Student 45 min

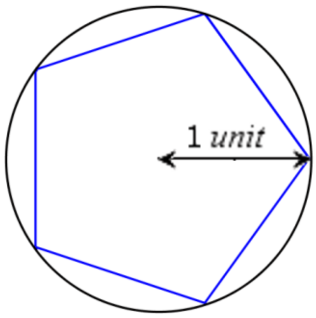
Introduction



Suppose you didn't know the formula for the circumference of a circle, you could approximate the circumference by calculating the perimeter of a square that just fits inside the circle. A better approximation could be achieved by using a regular pentagon, and better again using a regular hexagon. This investigation explores the perimeter of a regular *n* sided polygon that just fits inside a circle with radius 1 unit.

Question: 1

Determine the perimeter of the square shown and state whether this approximation is too big or too small in relation to circumference of the circle.

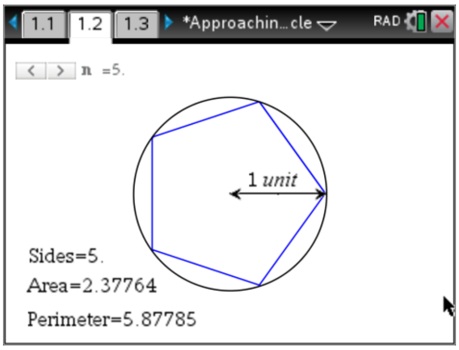


Question: 2

By dividing a regular pentagon into 10 right angled triangles, determine the perimeter of the pentagon and compare the result with the circumference of the circle that 'circumscribes' the pentagon.

Question: 3

By dividing a regular hexagon into 12 right angled triangles, determine the perimeter of the hexagon and compare the result the circumference of the circle that 'circumscribes' the hexagon.



Load the TI-Nspire file "Take it to the limit" and navigate to page 1.2. Use the slider to change the value of *n* and check your answers for the square, pentagon and hexagon.

Continue increasing the value of *n* and identify the limiting value for the perimeter of an *n* sided regular polygon.

Check the graph on page 1.3 that shows a record of the number of sides and the corresponding circumference.

Question: 4

What is the limiting value for the perimeter of an *n* sided regular polygon? Will this limit ever be reached or exceeded? Explain.

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Question: 5

Write a rule for the perimeter of an *n* sided regular polygon and define it as a function $f(n)$ in the calculator application on page 2.1. Use the 'limit' command in the calculus menu to determine the limit of your rule as $n \to \infty$. Note that the document is set to 'radians' so the rule, wherever trigonometric functions are involved, should take this into account.

Syntax: $\lim\limits_{n \to \infty} f(n)$

In this investigation *n* is used in the slider in problem 1 for the number of sides for the regular polygon. This means it is assigned a value, making it a parameter rather than a true variable. By using a **New Problem** within the document all links to *n* are cleared for problem 2 returning *n* to variable status. Adding problems to a document allows multiple uses of the same variable. The page number is now 2.1 signifying it is problem 2, page 1.

Question: 6

Another way to approximate the circumference of the circle is to put a regular polygon around the outside of the circle.

a. Determine the perimeter of a square that just fits around the outside of the circle.

b. Determine the perimeter of regular pentagon that just fits around the outside of the circle.

c. Define a rule for an *n* sided regular polygon and determine the limit of this function as $n \to \infty$

d. Will the perimeter of the outer polygon ever equal the circumference of the circle?

Page 3.1 contains a dynamic representation of the inner and outer polygons, use this representation to check your equation for the perimeter of the polygon that just fits around the circle. The graph on page 3.2 shows how both functions approach the same limit, but from different directions.

Question: 7

The polygons can be used to approximate the area of the circle. Page 3.1 includes a dynamic representation of this including the corresponding values for the area. The area produced by the inner (inscribed) polygon produces a smaller estimate for the area. The outer (circumscribed) polygon produces a larger estimate. Determine the respective rules for the area of the inscribed and circumscribed polygons and show that they approach the same limit as $n \to \infty$.

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