Permutations

ID: 10076

Time required
35 minutes

Topic: Permutations, Combinations & Probability

- *Use the Fundamental Counting Principle to calculate the number of outcomes in a sample space.*
- *Use factorial notation to express the number of permutations and combinations of n elements taken r at a time.*
- *Use factorial notation to express the number of outcomes in a sample space.*
- *Evaluate expressions involving factorials to compute the number of outcomes in a sample space.*

Activity Overview

This activity is an introduction to permutations. It includes an optional review on factorials and the Fundamental Counting Principle (also called the Basic Counting Rule). Students are then led through the development of the formula for finding n objects taken n at a time and n objects taken r at a time. They are given several problems to solve. Lastly, an optional extension allows students to use the formula for permutations with repetition.

Teacher Preparation

This activity is appropriate for an Algebra 2 or Precalculus classroom.

- *This activity is designed to be used in an Algebra 2 classroom. It can also be used in an introductory Statistics course or an advanced Algebra 1 course.*
- *Problem 1 is an introductory question and Problem 2 is a review of factorials and the Fundamental Counting Principle. Problem 2 can be skipped if this review is not needed in your class.*
- *Many students struggle with permutations and combinations. For this reason, it is important that you do not rush through the activity or assume that any part of the activity is "too easy." A solid foundation on these basic concepts will help them later when the problems become more complex.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "10076" in the quick search box.***

Classroom Management

- *This activity is intended to be mainly **teacher-led**, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their calculators.*
- *The student worksheet helps guide students through the activity and provides a place for students to record their answers.*
- *Information for an optional extension is provided at the end of this activity, both on the student worksheet and in this document. Should you not wish students to complete the extension, you may have students disregard that portion of the student worksheet.*

TI–84 Plus Applications

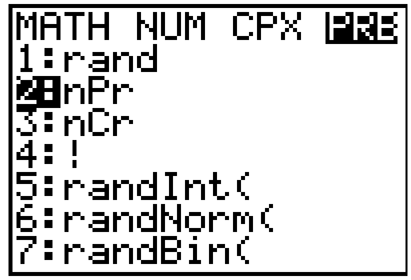
none

Permutations

ID: 10076

In this activity, you will explore:

- *graphing an ellipse using a Cartesian equation*
- *finding the parametric equations for an ellipse*
- *modeling the orbit of Jupiter*



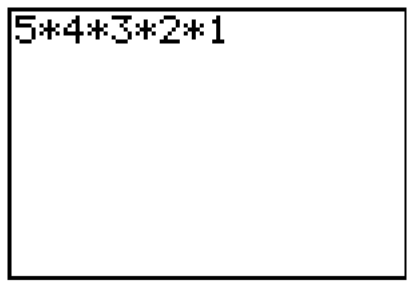
Problem 1 – An introduction

Use your best judgment to answer the question on your worksheet. Use your calculator for any necessary calculations. You will see it again at the end of the activity. After working through the activity, this question should be easy!

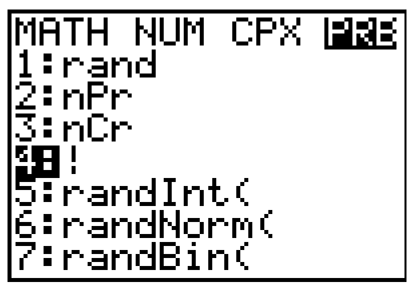
Problem 2 – Factorials and the Fundamental Counting Principle

Use your calculator to evaluate each expression:

- 5 · 4 · 3 · 2 · 1
- 5!



To type the factorial symbol **!**, go to MATH > PRB and choose it from the menu.



Discuss the definition of the factorial of a number with your class. Calculate more factorials to test the definition.

Evaluate each expression:

- 0!
- (5 – 2)!
- 5! – 2!

Discuss with your class whether the following statement is true or false.

(5 – 2)! = 5! – 2!

A spinner with four equal sections colored red, green, blue, and yellow is spun, and a penny is flipped. List all possible outcomes on your worksheet.

Discuss the following questions with the class:

- How many outcomes are there?
- What multiplication expression could have been used to find the answer?
- How the list would change if there were 10 colors on the spinner?

A penny is flipped three times. List all possible outcomes on your worksheet.

Discuss the following questions with the class:

- How many outcomes are there?
- What multiplication expression could have been used to find the answer?
- How the list would change if the penny were flipped four times?

State the Fundamental Counting Principle in your own words on your worksheet.

Problem 3 – *n* objects taken *n* at a time

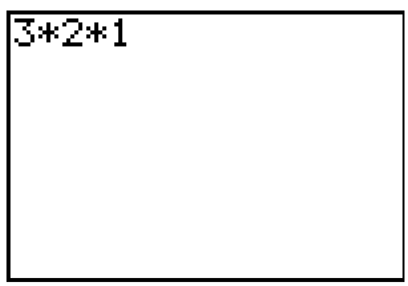
List all the ways in which the letters *a*, *b*, and *c* can be arranged on your worksheet.

Because you are looking for every possible arrangement, you are finding the number of ***permutations***.

A permutation is an *arrangement* of objects in which order counts. When finding permutations, a different order is a different permutation. Therefore, *a*, *b*, *c* is a different permutation than *a*, *c*, *b* even though they contain the same letters.

Write a multiplication expression that can be used to find the answer on your worksheet. Use the following as a guide:

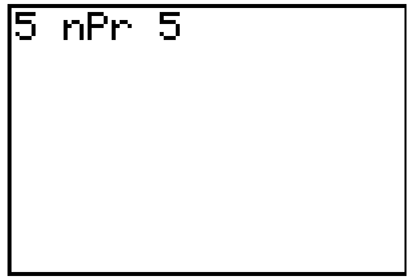
choices for 1st spot × # choices for 2nd spot × # choices for 3rd spot



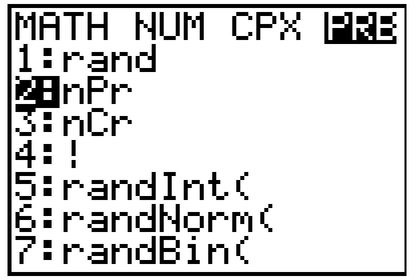
Discuss with the class: how can you write this expression as a factorial?

You have found the number of permutations of *n* items taken *n* at a time (meaning all available items were used). This can be denoted as ${}_nP_n$. In general, permutations are shown as ${}_nP_r$, where *n* is the number of available items and *r* is how many of those items are actually being arranged. In the previous example, *r* happens to equal *n*.

You can use the **nPr** command to find the number of permutations. The screenshot shows how to calculate the number of arrangements of 5 items chosen from 5 available items.



To type the **nPr** command, go to MATH > PRB and choose it from the list.



- Use the **nPr** command to find all the ways in which the letters *a*, *b*, and *c* can be arranged.
- Use the **nPr** command to find how many different ways you can arrange the letters in the word **NUMBER**.

Note that it is *not* always easy to write out all arrangements by hand and count them!

Discuss your answers with the class. Where have you seen these numbers before? Then complete the equation ${}_nP_n = \square$ on your worksheet.

Evaluate a few more permutations where the number of items being arranged is equal to the number of available items to test your equation.

Problem 4 – *n* objects taken *r* at a time

List all of the ways to arrange two of four letters on your worksheet.

Write a multiplication expression that can be used to find the answer on your worksheet.
Use the following as a guide:

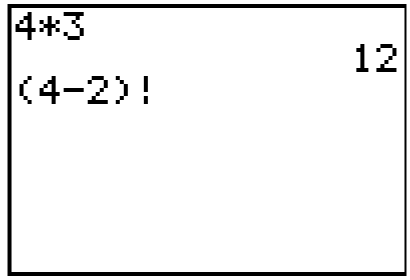
$$\underline{\text{\# choices for 1st spot}} \times \underline{\text{\# choices for 2nd spot}}$$

Discuss with your class whether this expression could be represented as a factorial. Why or why not?

What multiplication expression should be written in the denominator to make the following equation true?

$$\frac{4 \cdot 3 \cdot 2 \cdot 1}{\square} = 12$$

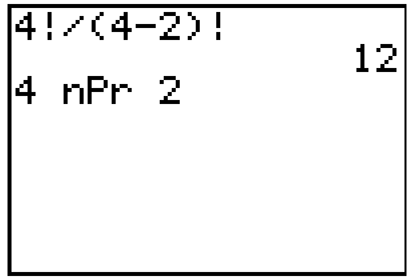
Once you have completed it, write the expression above using factorial notation. Use your calculator to check. Discuss with your class how to write (4 – 2)! as a single factorial.



The formula for finding the number of permutations of *n* unique objects taken *r* at a time is:

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Confirm the answer to the previous question by using the permutation command.



Use both the definition of $_{n}P_{r}$ and the $_{n}P_{r}$ function on the handheld to answer the following.

A collector has 16 statues. In how many ways can the collector arrange 5 of the statues on a shelf?

Discuss your answer with the class. Which is easier, to use formulas to calculate the number of arrangements or to list and count them by hand?

Problem 5 – Practice

Answer the questions in Problem 5 on your worksheet on your own. Review the answers with your class.

Extension

The formula $_{n}P_{r} = \frac{n!}{(n-r)!}$ only applies to arrangements of n unique objects.
The formula for permutations with repetitions is different.

The number of different permutations of *n* objects, of which q_1 are of one kind, q_2 are of a second kind, ...q_k are of a *k*-th kind is given by

$$\frac{n!}{(q_1)!(q_2!)..(q_k!)}$$

Use this formula to answer the questions on your worksheet.

Solutions

Problem 1

- C

Problem 2

- $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$
- 5! = 120
- 0! = 1
- (5 – 2)! = 6
- 5! – 2! = 118
- r, h r, t g, h g, t b, h b, t y, h y, t
- hhh, hht, hth, htt, thh, tht, tth, ttt
- All possible outcomes in a sample space can be found by multiplying the number of ways each event can occur.

Problem 3

- abc, acb, bca, bac, cab, cba
- 3 · 2 · 1
- *n*!
- 720

Problem 4

- a, b a, c a, d b, a b, c b, d c, a c, b c, d d, a d, b d, c
- 4 · 3
- ${}_nP_r = \frac{n!}{(n-r)!}$
- 524,160

Problem 5

- ${}_nP_r(26,5) = 7{,}893{,}600$ (intro question)
- ${}_nP_r(6,6) = 720$
- ${}_nP_r(10,3) = 720$
- ${}_nP_r(26,3) \cdot {}_nP_r(10,5) \cdot 2 = 943{,}488{,}000$

Extension

PIZZA: 60; SUCCESS: 420: COOKBOOK: 840; and MISSISSIPPI: 34,650