Transformational Geometry Dilations
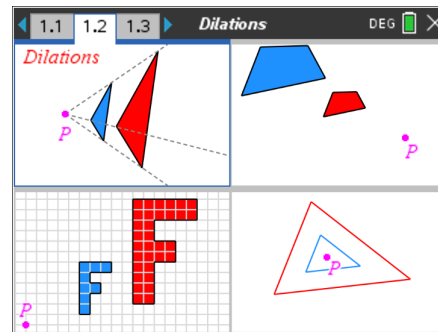**Student Activity**

Name _____

Class _____

Transformational Geometry is a way to study geometry by focusing on geometric "movements" or "transformations" and observing/studying properties about these figures.

There are four geometric transformations:
< Reflections   < Translations   < Rotations   **< Dilations**

Play - Investigate - Explore - Discover
P I E D

Download and install the red TI-Nspire student software and the Dilations TNS file from the website where you obtained this document.

Then you can interact with these figures, too. If you decide not to download the software, or if you cannot, you can still do this activity along with the videos.

1. What do the 4 parts of the screen have in common in the screen shot above?
 Make at least two conjectures.   (a **conjecture** is an opinion or conclusion based upon what is observed.)

In the figure to the right, ΔABC is dilated about point P with a scale factor of 2.
ΔABC is called the pre-image while $\Delta A'B'C'$ is called the image (of dilation).
$\Delta A'B'C'$ is read "triangle A prime, B prime, C prime."
Point P is called the point of dilation.
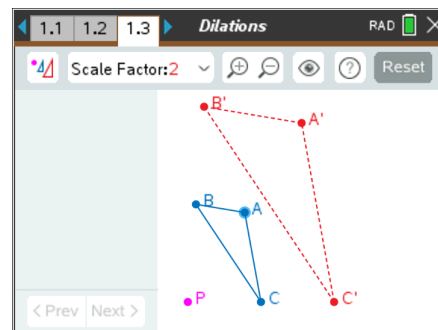
2. Using the TI-Nspire technology, dilate ΔABC about point P using a Scale Factor of 2.
Make a conjecture about what seems to be true about ΔABC and its image, $\Delta A'B'C'$.

3. a. Dilate $\triangle ABC$ about point P using a Scale Factor of 3. Zoom ⊖ (out) as needed.
P I E D

b. Dilate $\triangle ABC$ about point P using a Scale Factor of $\dfrac{1}{2}$ (1/2). P I E D

4. Dilate $\triangle ABC$ about point P using a Scale Factor of 1.5. Zoom ⊖ (out) as needed.

a. Grab and pull point P to coincide with vertex A. What two segments appear to be parallel?

b. Grab and pull point P to coincide with vertex B. What two segments appear to be parallel?

c. Before you grab and pull point P to coincide with vertex C, what two segments would you expect to appear to be parallel?

d. Now grab and pull point P to coincide with vertex C. What two segments appear to be parallel?

5. Dilate $\triangle ABC$ about point P using a Scale Factor of – 1.
Play – Investigate – Explore – Discover.
How would you describe the relationship between the pre-image and the image?

Angles and Sides
Note: the measurements given using TI-Nspire technology are rounded to the nearest hundredth.

6. Dilate $\triangle ABC$ about point P using a Scale Factor of 1.5. Zoom ⊖ (out) as needed.
P I E D.
a. What appears to be true about the angles of the two triangles?

b. Do the corresponding sides have the same lengths?

7. Tools > Templates > 1. Angles & Sides
Dilate $\triangle ABC$ about point P using a Scale Factor of 1.5. Zoom ⊖ (out) as needed.
a. What is now shown on the screen?

P I E D.
b. Make a conjecture about what you observe?

c. Let's make sure our conjecture holds true for other Scale Factors.

Repeat parts a and b above but with a Scale Factor of $\dfrac{1}{2}$. P I E D.

Does your conjecture still seem to be true?

8. Tools > Templates > 1. Angles & Sides

⊖

Dilate $\triangle ABC$ about point P using a Scale Factor of 2. Zoom   (out) as needed.

a. Click on   Next >

Look at the lengths of corresponding sides of the two triangles.
P I E D.
Note: we suggest that you try to make one or two of the side lengths a whole number.

Make a conjecture about what appears to be true about the lengths of corresponding sides of the two triangles.

b. Without pressing Reset, change the Scale Factor to 3.

Look at the lengths of corresponding sides of the two triangles.
P I E D.
Note: we suggest that you try to make one or two of the side lengths a whole number, if possible.

Make a conjecture about what appears to be true about the lengths of corresponding sides of the two triangles.

c. Click on   Next >

What is shown on this screen?
P I E D.

Does this screen validate your conjecture?

d. Without pressing Reset, change the Scale Factor back to 2.

Look at the lengths of corresponding sides of the two triangles.
P I E D.

Make a conjecture about what appears to be true about the lengths of corresponding sides of the two triangles.

e. What would you expect to observe if the Scale Factor were to be $\dfrac{1}{2}$?

Use the technology to see if you are correct.

Perimeters and Areas

Note: the measurements given using TI-Nspire technology are rounded to the nearest hundredth.

9. Tools > Templates > Perimeters and Areas
Dilate $\triangle ABC$ about point P using a Scale Factor of **2**. Zoom   (out) as needed.

Grab and move one of the vertices of $\triangle ABC$ until the **perimeters are whole numbers, if easily possible. If not, just use the values from the video.**

a. Record these values into the Perimeter Table below, a. Scale Factor = **2** column.
Notice that the image perimeter is listed first in the column, the pre-image perimeter below it.
Look for patterns as you grab and move vertices of the pre-image triangle.

Perimeter Table

In Units	a. Scale Factor 2	b. Scale Factor 3	c. Scale Factor $\dfrac{1}{2}$
Perimeter of image $\triangle A'B'C'$			
Perimeter of pre-image $\triangle ABC$			

b. Reset. Dilate $\triangle ABC$ about point P using a Scale Factor of **3**. Zoom   (out) as needed.

Grab and move one of the vertices of $\triangle ABC$ until the **perimeters are whole numbers, if possible**.
If not easily possible, just use the values from the video.
Place these values into the Perimeter Table above, b. Scale Factor = **3** column.

c. Reset. Dilate $\triangle ABC$ about point P using a Scale Factor of $\dfrac{1}{2}$. Zoom as needed.

Grab and move one of the vertices of $\triangle ABC$ until the **perimeters are whole numbers, if possible**.

Place these values into the Perimeter Table above, c. Scale Factor = $\dfrac{1}{2}$ column.

d. Look at each pair of corresponding perimeters in the columns. What pattern do you see?

10. Let's continue to use TI-Nspire technology to discover or confirm the pattern.
a. Reset. Dilate $\triangle ABC$ about point P using a Scale Factor of **2**. Zoom as needed.

b. Click on   Next >   twice to see **ratios of the perimeters of the image to the pre-image triangle**.

Grab and move the vertices of $\triangle ABC$ and notice the perimeters change. But what does **NOT** change?

c. Notice that the 4th (last) row of the Perimeter Table is empty.
Into the first column of the 4th row, write the following:

$$\frac{Perim\left(\triangle A'B'C'\right)}{Perim\left(\triangle ABC\right)}$$

d. Into the last row of the column marked "**a.** Scale Factor **2**", write the ratio of the perimeters, image to pre-image.

e. Change the scale factor to **3**. Zoom as needed. Grab and move vertices and observe.
Into the last row of the column marked "**b.** Scale Factor **3**", write the ratio of the perimeters, image to pre-image.

f. Change the scale factor to $\frac{1}{2}$. Zoom as needed. Grab and move vertices and observe.

Into the last row of the column marked "**c.** Scale Factor $\frac{1}{2}$", write the ratio of the perimeters, image to pre-image.

11. a. Reset. Change the Scale Factor to be **3**.

Dilate $\triangle ABC$ about point P using a Scale Factor of **3**. Zoom ⊖ (out) as needed.

Click on Next > to see the **areas** displayed.

Grab and move vertices until the area of the pre-image $\triangle ABC$ is a multiple of ten.

Record these values into the Area Table on the top of the next page, **a**. Scale Factor = **3** column.

Notice that the image area is listed first in the column, the pre-image area below it.

Look for patterns as you grab and move vertices of the pre-image triangle.

Area Table

In Square Units	a. Scale Factor 3	b. Scale Factor 2	c. Scale Factor $\dfrac{1}{2}$
Area of image $\Delta A'B'C'$			
Area of pre-image ΔABC			

b. Reset. Dilate ΔABC about point P using a Scale Factor of **2**. Zoom as needed.
Grab and move one of the vertices of ΔABC until the **areas are multiples of ten**.
Place these values into the Area Table above, **b**. Scale Factor = **2** column.

c. Reset. Dilate ΔABC about point P using a Scale Factor of $\dfrac{1}{2}$. Zoom as needed.

Grab and move one of the vertices of ΔABC until the **areas are multiples of ten**.

Place these values into the Perimeter Table above, **c**. Scale Factor = $\dfrac{1}{2}$ column.

d. Look at each pair of areas in the columns. What pattern do you see?

12. Let's continue to use TI-Nspire technology to discover or confirm the pattern.
a. Reset. Dilate ΔABC about point P using a Scale Factor of **3**. Zoom as needed.

Click on   Next >   three times to see the **ratios of the areas of the image to the pre-image triangle**.
Grab and move the vertices of ΔABC and notice the areas change. But what does **NOT** change?

b. Notice that the 4th (last) row of the Area Table is empty.
Into the first column of the 4th row, write the following:

$$\dfrac{Area\left(\Delta A'B'C'\right)}{Area\left(\Delta ABC\right)}$$

c. Into the last row of the column marked "**a.** Scale Factor **3**", write the ratio of the areas, image to pre-image.

d. Change the scale factor to **2**. Zoom as needed. Grab and move vertices and observe.
Into the last row of the column marked "**b.** Scale Factor **2**", write the ratio of the areas, image to pre-image.

e. Change the scale factor to $\dfrac{1}{2}$. Zoom as needed. Grab and move vertices and observe.

Into the last row of the column marked "**c.** Scale Factor $\frac{1}{2}$", write the ratio of the areas, image to pre-image.

13. Let's a) summarize with a specific number and then b) generalize with a variable.
a. Suppose $\triangle ABC$ is dilated about a point with a Scale Factor of 4.
Based upon what we have observed, complete the following:

$$\frac{Perimeter\left(\triangle A'B'C'\right)}{Perimeter\left(\triangle ABC\right)} = \underline{\hspace{1.5cm}} \qquad\qquad \frac{Area\left(\triangle A'B'C'\right)}{Area\left(\triangle ABC\right)} = \underline{\hspace{1.5cm}}$$

b. Suppose $\triangle ABC$ is dilated about a point with a Scale Factor of some number, R.
Based upon what we have observed, complete the following:

$$\frac{Perimeter\left(\triangle A'B'C'\right)}{Perimeter\left(\triangle ABC\right)} = \underline{\hspace{1.5cm}} \qquad\qquad \frac{Area\left(\triangle A'B'C'\right)}{Area\left(\triangle ABC\right)} = \underline{\hspace{1.5cm}}$$

Grid and Coordinates
14. Tools > Templates > Grid & Coordinates
Dilate $\triangle ABC$ about point P using a Scale Factor of **2**. Zoom as needed.

a. Observe the coordinates of the vertices of the pre-image triangle and the image triangle and look for patterns. Record the coordinates into the Table on the next page in the 'Figure 1' row.
b. Grab and move points A, B, C. Record the coordinates into the 'Figure 2' row. Pattern?

c. Repeat: Grab and move points A, B, C. Record the coordinates into the 'Figure 3' row. Pattern?

d. Now grab and move point P and record the coordinates into the 'Figure 4' row. Pattern?

Scale Factor = **2**	A	B	C	A'	B'	C'
Figure 1						
Figure 2						
Figure 3						
Figure 4 (move P)						

e. Look at the Figure 1, 2, and 3 rows. Make a conjecture about the coordinates of the vertices of a triangle and its image under a dilation about the origin.

f. Does your conjecture still hold true if the dilation point P is not at the origin (Figure 4)? Explain.

15. Let's repeat the investigation we did in number 14 above, but with a different scale factor, $\frac{1}{2}$.

Dilate $\triangle ABC$ about point P using a Scale Factor of $\frac{1}{2}$. Zoom as needed.

a. Observe the coordinates of the vertices of the pre-image triangle and the image triangle and look for patterns. Record the coordinates into the Table on the next page in the 'Figure 1' row.

b. Grab and move points A, B, C. Record the coordinates into the 'Figure 2' row. Pattern?

c. Repeat: Grab and move points A, B, C. Record the coordinates into the 'Figure 3' row. Pattern?

d. Now grab and move point P and record the coordinates into the 'Figure 4' row. Pattern?

Scale Factor = $\frac{1}{2}$	A	B	C	A'	B'	C'
Figure 1						
Figure 2						
Figure 3						
Figure 4 (move P)						

e. Look at the Figure 1, 2, and 3 rows. Make a conjecture about the coordinates of the vertices of a triangle and its image under a dilation about the origin.

f. Does your conjecture still hold true if the dilation point P is not at the origin (Figure 4)? Explain.

16. Summary and generalization.

a. Suppose $\triangle DEF$ is dilated about the origin using a Scale Factor of **3**. The coordinates of the

pre-image triangle are listed below. Based upon what you have observed, list what you expect the

coordinates of the image triangle to be.

Scale Factor = **3**	$D:\ (1,\ 4)$	$E:(-2,\ 5)$	$F:(-2,\ -6)$
	$D':$	$E':$	$F':$

b. Suppose $\triangle XYZ$ is dilated about the origin using a Scale Factor of **R**.

The coordinates of point X are $(a,\ b)$. What do you expect the coordinates of $X\,'$ to be?

Distance from P to the Vertices

17. Tools > Templates > Dist P to Vertices

a. Dilate $\triangle ABC$ about point P using a Scale Factor of **2**. Zoom as needed.

Investigate the distances from P to each of the 6 vertices by grabbing and moving each of the vertices of $\triangle ABC$. Try to make as many distances as possible to be whole numbers. Look for patterns.

b. Record the 6 lengths below.

$Scale\ \ Factor\ =\ 2 \quad\quad PA\ = \quad\quad\quad\quad PB\ = \quad\quad\quad\quad PC\ =$

$PA'\ = \quad\quad\quad\quad PB'\ = \quad\quad\quad\quad PC'\ =$

c. Make a conjecture based on what you have observed.

d. Click on $\boxed{\text{Next} >}$. Observe.

Grab and move each of the vertices of $\triangle ABC$ and point P.

Does this seem to validate your conjecture?

e. Change the Scale Factor to 3 and observe what happens as you grab and move the vertices and P.

f. Change the Scale Factor to $\dfrac{1}{2}$ and observe what happens as you grab and move the vertices and P.

g. **Summary.**

Given: $\triangle DEF$ is dilated about point P with a Scale Factor of 4. Complete the following:

If $PD = 3$, then $PD' =$ If $PE = 6.2$, then $PE' =$ If $PF = 8.7$, then $PF' =$

18. Tools > Templates > Slopes

Click on the   ⊙   menu and click on Grid.

a. Dilate $\triangle ABC$ about point P using a Scale Factor of **2**. Zoom as needed.
Observe the slopes of corresponding sides.
Grab and move the 3 vertices and continue to observe the slopes of corresponding sides.
Make a conjecture.

And what does that mean about the corresponding sides?

b. Change the Scale Factor to 3. Zoom as needed.
Observe the slopes of corresponding sides.
Grab and move the 3 vertices and continue to observe the slopes of corresponding sides.
Make a conjecture.

And what does that mean about the corresponding sides?

c. Change the Scale Factor to $\dfrac{1}{3}$.
Do a similar investigation as you did in parts a and b.

d. **Summary.**
Given: $\triangle DEF$ is dilated about point P with a Scale Factor of 4.

State 3 pairs of segments that are parallel.

19. **Dilate $\triangle ABC$ about point P with a Scale Factor of 2 by hand.**
We suggest that you do this along with the video and pause as needed.

You will need a compass and straightedge.

If you don't have a compass, do the best that you can without one.

If you have TI-Nspire technology, you can use the Dilations TNS file, pages 2.1 – 2.3.

It will take you through the steps at your own pace.