Thursday Night Precalculus Series
February 8, 2024

In this *AP Precalculus Live* session, we will explore several examples on solving trigonometric equations and inequalities using both a restricted domain and finding all solutions. We will rewrite trigonometric expressions using trigonometric identities.

About the Lesson

- This Teacher Notes guide is designed to be used in conjuction with the AP Precalculus Live session and Student Problems document that can be found on-demand: **https://www.youtube.com/watch?v=D61KxiGWlwg**
    - *Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the "Live Session" but can be enriched by that resource.*
- This session involves solving trigonometric equations and inequalities. It also involves rewriting trigonometric expressions in equivalent forms.
- The trigonometric identities used include:
    - The Pythagorean identities,
    - The sum and difference identities for sine and cosine,
    - The double angle identities for sine and cosine.
- Students should be able to use the TI-Nspire to check solutions to equations and inequalities as well as confirm the equivalence of representations of trigonometric functions.
- **Class Discussion**: Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

Materials:

TI-Nspire document
- Trig_Eq,_Ineq,_Identities.tns

Student document
- Problems_02_08

Solutions
- Precal_problems_solutions_02_08

YouTube
- https://www.youtube.com/watch?v=D61KxiGWlwg

AP Precalculus Learning Objectives

- 3.10.A: Solve equations and inequalities involving trigonometric functions.
- 3.12.A: Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.
- 3.12.B: Rewrite trigonometric expressions in equivalent forms with sine and cosine sum identities.

- 3.12.C: Solve equations using equivalent analytic representations of trigonometric functions.

Source: AP Precalculus Course and Exam Description, The College Board

Problem 1.

(a) Find all the values of x that satisfy the equation $\sqrt{2}\cos(4x)+1=0.$

(b) Find all the values of x in the interval $0 \le x \le \pi$ that satisfy the inequality $\sqrt{2}\cos(4x)+1<0.$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Teacher Tip: Students need practice with solving equations or inequalities with x as the argument of the trigonometric function. Students should then progress to equations or inequalities with $a \cdot x$ as the argument, such as the equation in 1. (a).

Class Discussion:

We have two intervals as solutions to the inequality when the closed interval is $0 \le x \le \pi.$ *If the closed interval is changed to* $0 \le x \le 2\pi,$ *how many intervals would we have as solutions to the inequality?*

Possible Answers: There would be four intervals as solutions to the inequality if the closed interval is now $0 \le x \le 2\pi.$

Class Discussion:

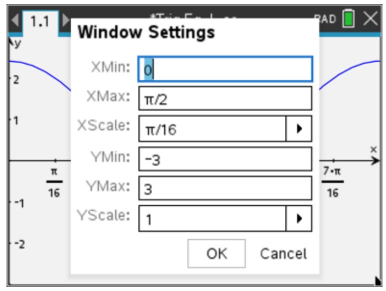
What is the period of the function $f(x)=\sqrt{2}\cos(4x)+1?$

Possible Answers: The period is $\frac{\pi}{2}.$

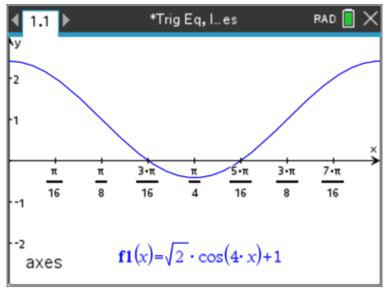
Teacher Tip: Revisit these discussions as we work through the graphing calculator solutions.

Graph the function $f(x)=\sqrt{2}\cos(4x)+1$.

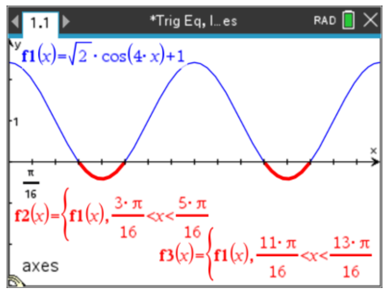
Use the Window Settings shown to the right. If we set the xScale to $\frac{\pi}{16}$, we can see that the two zeros of the function in the interval $0 \le x \le \pi$ are $\frac{3\pi}{16}$ and $\frac{5\pi}{16}$.



Technology Tip: It is possible to set the attributes on the x-axis to show the values at the tick marks as shown to the right.



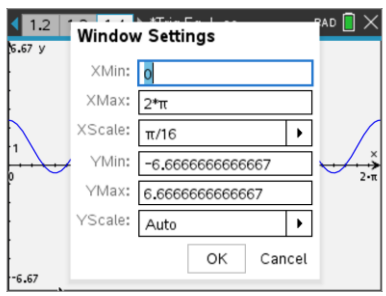
To verify the solutions to the inequality in 1. (b), graph the trigonometric function $f(x)=\sqrt{2}\cos(4x)+1$ on the given closed interval $0 \le x \le \pi$. Use piecewise functions as shown to verify that the solution to the inequality consists of two intervals.



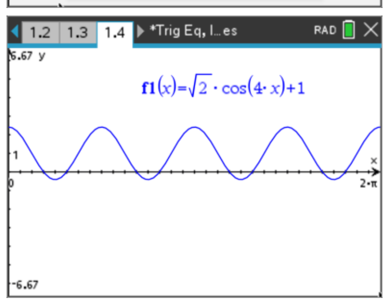
Review the Class Discussion since the graph confirms the four intervals on $0 \le x \le 2\pi$.

Class Discussion:

We have two intervals as solutions to the inequality when the closed interval is $0 \le x \le \pi$. If the closed interval is changed to $0 \le x \le 2\pi$, how many intervals would we have as solutions to the inequality?



Possible Answers: There would be four intervals as solutions to the inequality if the closed interval is now $0 \le x \le 2\pi$.



Problem 2.

(a) Find all the values of x that satisfy the equation $\frac{1}{\sqrt{3}}\sin(2x)-\frac{1}{2}=0.$

(b) Find all the values of x in the interval $0 \le x \le 2\pi$ that satisfy the inequality $\sin(2x) < \cos x.$

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

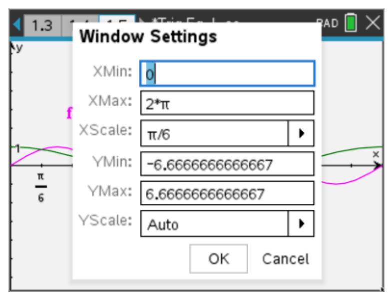
Class Discussion:

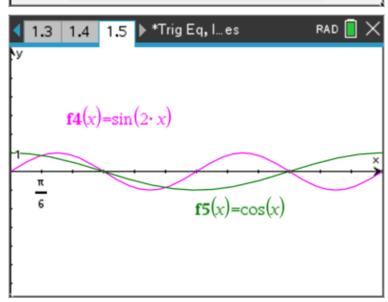
In 2. (b) analytical solution of $2\sin x\cos x < \cos x,$ *why can't we divide both sides by* $\cos x$?

Possible Answers: One issue is that there are values of x for which $\cos x = 0.$ We also want to use the zero product property with sign charts.

Teacher Tip: Sign charts are helpful with solving inequalities.

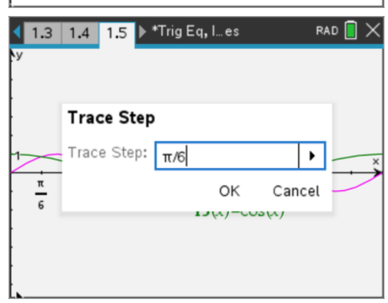
Use the graphing application to confirm the solution. Graph the two functions on windows shown to the right. Use an XScale of $\frac{\pi}{6}$.





Trace will also be useful to check the intervals.

Technology Tip: Set the Trace to $\frac{\pi}{6}$.



Problem 3.

What are all the values of $\theta,\ 0 \le \theta \le \pi,$ for which $2\sin(2\theta) \ge 1$ and $2\cos\theta \ge 1$?

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Class Discussion:

In previous problems, we added 2π to the endpoints for an additional interval. Why didn't we do that here?

Possible Answers: We have a restricted domain. We can check the interval where 2π was added to the endpoints to obtain $\frac{13\pi}{12} \le \theta \le \frac{17\pi}{12}$. This interval is not in the given domain.

Problem 4.

(a) Rewrite as an expression in which $\cos x$ appears once and no other trigonometric functions are involved.

$$\frac{1}{1-\sin x}+\frac{1}{1+\sin x}$$

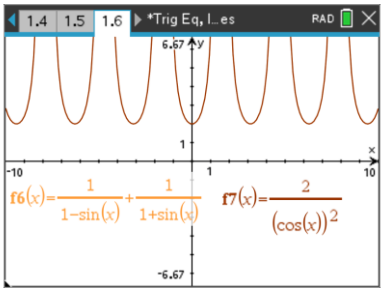
(b) Rewrite as an expression in which $\sin x$ appears once and no other trigonometric functions are involved.

$$3\sin x - 4\sin^3 x$$

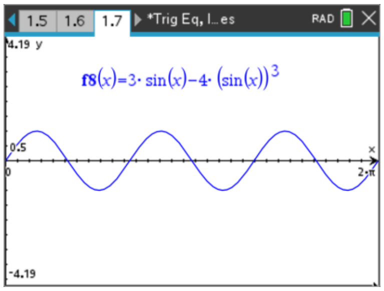
Sample Solution:

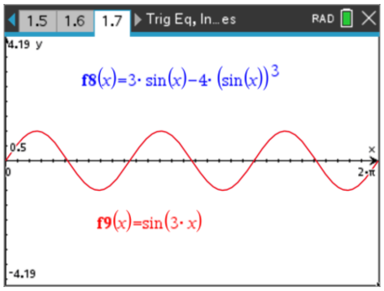
Refer to the Teacher Solutions Document for the full solution to this problem.

For 4. (a), graph the initial expression and the rewritten expression to verify the solution. Use a Zoom Trig window. The two graphs should match.



For 4. (b), use Xmin = 0 and Xmax = 2π.





Class Discussion:

In 4. (b) we only graphed the first function. How many cycles do we see in the interval from x = 0 to x = 2π? How could this information be used to determine the sin function that is represented in the rewrite?

Possible Answers: There are three (3) cycles shown in the interval from x = 0 to x = 2π. The sine function would be $y = \sin(3x)$. Graphing would confirm.

Note: *The following problems, 5 and 6, are not discussed in the video.*

Problem 5.

Suppose $\sin x = \frac{1}{3}$ and $\cos y = \frac{1}{4}$, where x and y are in the interval $\left(0, \frac{\pi}{2}\right)$. Evaluate the expression $\sin(x - y)$.

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Problem 6.

The function f is given by $f(x) = \cos(2.5x - 0.15)$. The function g is given by $g(x) = f(x - 0.5)$. What are the zeros of g on the interval $0 \le x \le \pi$?

Sample Solution:

Refer to the Teacher Solutions Document for the full solution to this problem.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The graphing application can be used to verify solutions to both equations and inequalities.
- The calculator application can be used to solve equations.
- The graphing application is useful in verifying equivalence of trigonometric expressions.

For more videos from the AP Precalculus Live series, visit our playlist
https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI