Greedy Pig

Teachers Teaching with Technology™
Professional Development from Texas Instruments

Teacher Notes & Answers

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TI-Nspire™ Investigation Student 50 min

Introduction

This is a simple game involving a single die. Roll the die and score the number appearing uppermost. Continue rolling and progressively adding each roll to your total score; beware of the dreaded two (2)! If you roll a two, your score is returned to zero and the game ends. You can avoid the dreaded two by opting out of the game at any time. Opting out means that you bank your current score; your opponents may continue to accumulate points or be promptly returned to zero if a two appears. Each game ends when a two is rolled.

The overall aim is to be the "Series" winner. The series winner is the person with the highest average score for a nominated quantity of games. It is possible to win the series without obtaining the highest score in any individual games.

To help understand this game you will play a series of 10 games in groups (or as a class). While playing the 10 games, think about what strategy(s) might be important to obtain the highest average.

Teacher Notes:

To demonstrate the game, have all students stand. Roll a single die for the entire class and record the progressive outcomes and running total on the board for all students to see. Students can sit at any time and bank their score.

Comments:

- The whole class game ensures everyone understands how the scoring is done, however it can lead to more thought being given to the number of classmates left standing rather than thinking about the likely outcomes or expected return.
- One of the aims of this game is to establish the independent nature of each successive roll. Continually prompt students to discuss whether or not a two is likely with the next roll. Do NOT however say things such as "We must be due for a two soon" as this can reinforce incorrect concepts. It is okay to prompt "We haven't seen a two for a while, do you think we're likely to get one soon?"

Question: 1

Based on your first 10 games (series), identify some strategies that you adopted or would consider using to give you the best chance of obtaining the highest average score.

Answer: Answers will vary.

Teacher Notes: This question provides an opportunity for students to review their thinking but can also identify students that do not understand 'independent' events or students that are perhaps struggling to separate the risk (variation in expected return) for successive roles and 'independent' events. Students may also comment on the number of students standing (active) at any one time. From a teacher's perspective there are several things that may be observed during the whole class game:

- Boys tend to be higher risk takers and may be more likely to remain standing for reasons other than probability.
- At what point do most students opt out? "Wisdom of the crowd" is used my many industries to forecast, does the collective 'wisdom' of the class match the theoretical results?

Author: P. Fox

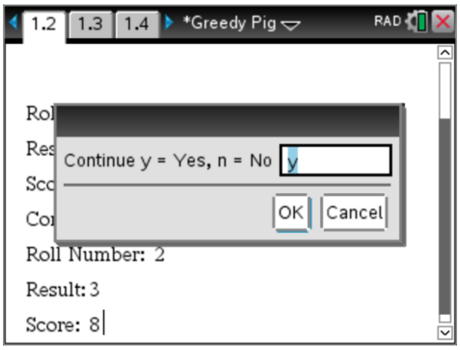
➢ Following individual games, consider asking students “why did you sit down when you did?” This is where you will also hear whether or not students believe that the events are independent.

Exploring and Simulating

Open the TI-Nspire document: **Greedy Pigs**.

Navigate to page 1.2 and select “Piggy1” from the variable [VAR] menu. By default the game selects “y” to continue. Press [enter] to continue in the game or “n” to cancel and bank your score.

The number of dice rolls, current roll (result) and progressive score (score) are displayed each time the die is rolled.



Teacher Notes: Students should seed their random number generator before starting the activity otherwise students may get the same sequence of ‘random numbers’. Of course it may be equally interesting to have all students ‘see’ the same random numbers without knowing what one another has on their screen.
To seed the random variable press: [Menu] > Probability > Random > Seed ... then type in a four (or more) digit number and press [enter]. Each student should chose a different seed value, or the same if every student is to receive the same sequence of ‘random’ numbers.

Question: 2

Record your results for 10 games in the table below. Your aim is to obtain the highest average for the 10 games.

Game:	1	2	3	4	5	6	7	8	9	10
No.Rolls										
Score:										

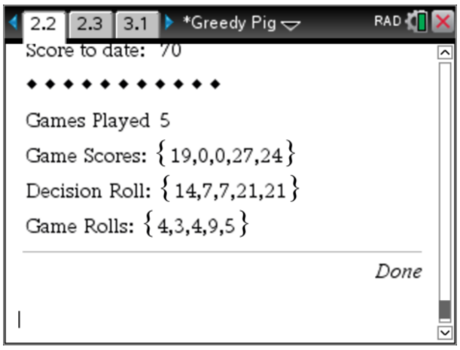
Question: 3

Compare your average score with some of the other students in your class. Did it make any difference to your strategy not being able to see your opponent’s scores? Explain.

Answers to the first part of this question will vary. (Average score) Students will most likely identify that their strategy was slightly different when they could not see their opponents’ progressive score. Students are more likely to focus on achieving a high overall score rather than for each specific game. They may however alter their strategy towards the end of the 10 games if they feel they have either achieved a particularly high score (reduce risk) or a particularly low score (increase risk).

Teacher Notes: Students responses should be used to drive discussion. Change in risk taking align to emotive neurological responses and is well understood by industries associated with gambling. The purpose of removing the visibility of opponent scores (calculator simulation) is to sharpen the focus back on mathematical strategies. Students may comment that they felt unlucky as they happened to get more two’s than others in the class. To overcome this argument, all students can use the same random seed value so the calculator will generate exactly the same sample to each student. Note that the random number sequence continues regardless of when the student decides to ‘bank’ their score. If the same sequence is required at the start of each game, students would need to re-seed the random number generator at the start of each game with each game having its own seed value.

Navigate to page 2.1 of the Greedy Pigs document.



The Piggy2 game prompts for the total number of games to be played. The game keeps record of some data that may be useful:

PS = Score for each game.

RS = Score immediately before the last role (when the decision was made to continue playing.

TS = Total number of dice rolls in the game.

When you're playing a game, it is easy to get caught up in the hype of 'just one more roll'. The limbic part of the brain deals with emotions and arousal but also runs interference in our ability to make logical decisions. Understanding how people make decisions is utilised by marketing companies, right down to where products are placed within supermarkets. The purpose of the Piggy 2 program is to allow you to explore strategies and at the same time recording your decision making process to ensure your decisions are based on specific strategies rather than emotional response.

The sample data above was for a person playing Greedy Pig in a 5 game series. The person entered with a strategy of banking the score as soon as they got to 15. Game one was successful, the decision role shows that the score got to 14, therefore 'play on', in the next roll the score was 19 so it was banked and the game finished. The second game shows that the score got to 7, decision was therefore to play on, a 2 was rolled sending the game score back to zero. This happened again in the third game. So far the player has stuck to their strategy. What happened in the fourth game? Chances are the person was concerned their average was going to be low due to the zero result for games two and three; they adjusted their strategy, increasing their risk, and choosing to play on when the score was 21. The game data shows this was probably an anxious time as they continued for a total of nine dice rolls. In game five we see that the strategy was not adhered to yet again, however this time the higher result came much quicker so less 'emotionally driven' decisions were made.

Question: 4

Write down a strategy for obtaining the best average for a 10 game series. Run the Piggy2 game and write down the three sets of results for PS, RS and TS.

Answer: Students answers will vary. Note that students are asked to write 'a strategy' so this question represents the current student's ideas about what might work.

Teacher Notes: Students that focus on the quantity of rolls most likely do not yet understand the independence of events. They may have some idea that you 'expect' to see a '2' approximately one in every six rolls of the die, however if a 2 hasn't appeared for 12 rolls, they 'expect' a 2 to come up soon. Students that are focusing on score have started to understand the level of risk associated with consecutive rolls, they may still hold the belief that if a 2 hasn't been rolled for a while ... it is more likely to come up next.

Question: 5

If you run the Piggy2 game again with exactly the same strategy, will you get the same results? Explain.

Answer: No. The events are based on chance so the chances of getting exactly the same outcome are very small.

Rolls vs Score – Collecting Statistical Evidence

Two common strategies to obtain the highest series average consist of specifying the number of rolls before banking the total (Rolls) or specifying the score before banking the total (Score). In this part of the Greedy Pig investigation you will explore these strategies by collecting data.

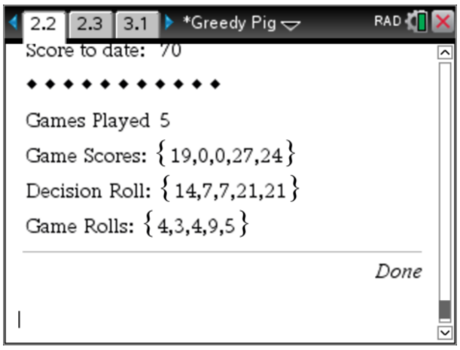
Author: P. Fox

Navigate to page 3.1 of the Greedy Pigs document and read the instructions, then navigate to page 3.2.

Piggy3 = Specify the number of rolls before banking the total. (Rolls)

Piggy4 = Specify the score before banking the total. (Score)

We will start by exploring the number of rolls strategy.



Question: 6

It is reasonable to believe that staying in the game for just one roll is unlikely to produce a competitive series average; similarly, holding out for 100 rolls is unlikely to produce a competitive series average. Think about what might be a reasonable quantity of dice rolls that provides a balance between obtaining a sufficient total each game versus the likelihood of getting a score of zero. Place this quantity in the grey square (below). Run the Piggy 3 program three times using your identified quantity of rolls. The Piggy 3 program will simulate 100 games and return the average (mean) result. Once you have finished exploring your selected quantity of rolls, explore quantities above and below your selected number of rolls and record these results.

Note: Each time the game is simulated, you can use other statistical measures to gain a greater insight as to what is happening. For the Piggy 3 simulation the number of rolls for each game is stored in "Rolls" and the game score in "Score".

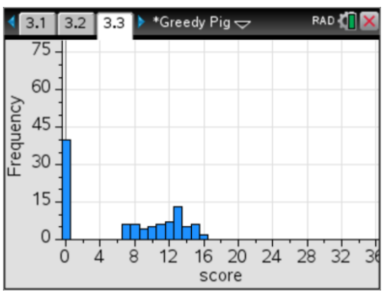
Sample Data:

No. Rolls	3	4	5	6	7	8	9
100 Game Average (Trial 1)	5.31	7.86	5.73	7.71	7.70	8.52	7.23
100 Game Average (Trial 2)	6.87	6.73	7.03	8.07	7.24	5.28	7.05
100 Game Average (Trial 2)	6.85	6.47	6.64	6.05	9.02	9.83	7.87

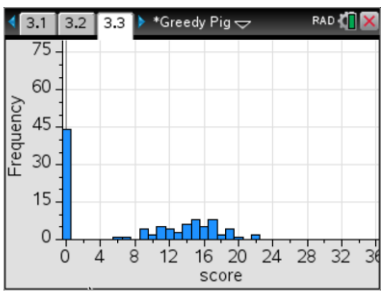
Answers: The individual data are not particularly enlightening. It is reasonable to conclude that stopping after three rolls provides the smallest return in this list. Students may however explore more extreme examples either side of their selected quantity.

Sample Graphs:

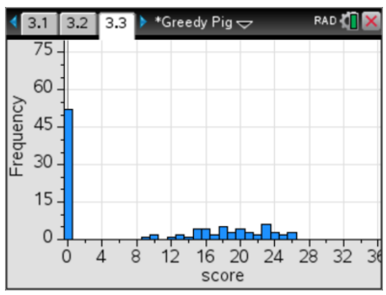
3 Rolls



4 Rolls



5 Rolls



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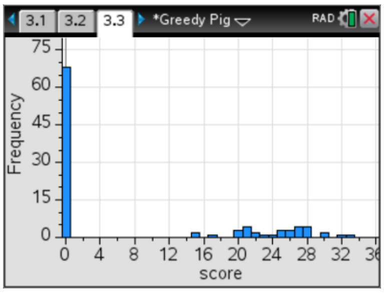
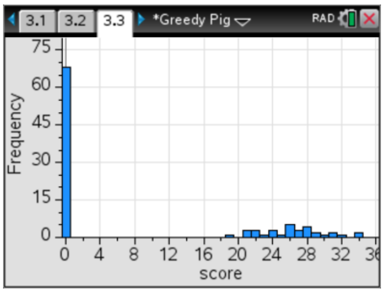
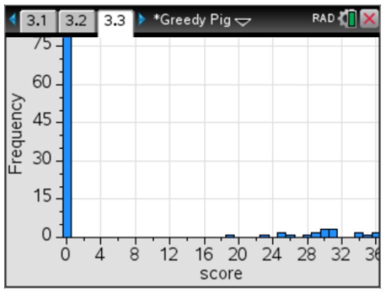
6 Rolls



7 Rolls



8 Rolls



Question: 7

Discuss the data that you have obtained. You can include graphs and other statistical measures in your discussion.

Answer: Students should notice that increasing the number of rolls increases the quantity of games with a zero score, at the same time, the successful scores become fewer in quantity but they move further to the right (larger scores) by approximately 3 units (19/6) with each additional roll. Students may not notice that the spread of results also increases. The increase in spread however is very logical as a 3 roll game could potentially include scores from 3 through to 18 whereas a 6 roll game could potentially include scores from 6 through to 36.

It is interesting to note that the median and mode for 5 or more rolls are both zero, so our selection of average to be the 'mean' provides a somewhat different view of the data.

Teacher: Students should set the viewing window before looking at all the graphs as the automatic scaling can conceal in part the natural increase in zero scores and the spread of data.
The bin width and bin alignment should also be set so that each graph shows specific scores rather than a selection of scores and the axis is straight forward being that numerical values displayed align with the centre of each column (bin).

Question: 8

In this question you will explore what happens if you target a specific score before banking your result.
Your score may be obtained in just 2 or 3 rolls (depending on your target) or it might take 10 rolls. Each game however continues until your target score is obtained.

Note: Each time the game is simulated, you can use other statistical measures to gain a greater insight as to what is happening. For the Piggy 3 simulation the number of rolls for each game is stored in "Rolls" and the game score in "Score".

Sample Data

Target Score	13	15	17	19	21	23	25
100 Game Average (Trial 1)	7.82	6.84	7.48	8.61	7.74	8.88	9.57
100 Game Average (Trial 2)	5.31	8.15	8.29	8.77	7.79	9.81	6.77
100 Game Average (Trial 2)	6.38	7.21	8.03	7.13	8.20	6.82	7.25

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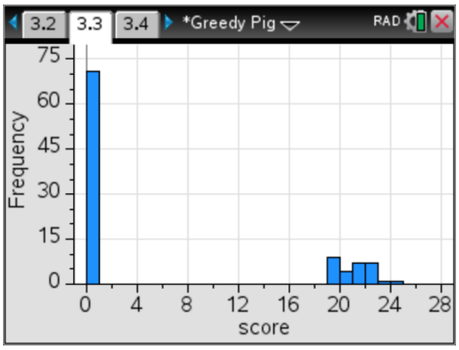
Answer: As per previous simulations, the data supports an approximate target value somewhere between 17 and 21. Students should note the variability in results particularly for the larger target scores. The graphs that students obtain for this data can be done as score frequency and/or rolls. The shape of the graphs is quite similar with the exception that for 'scores' the lower band of scores does not exist because the program does not stop if a low score exists.

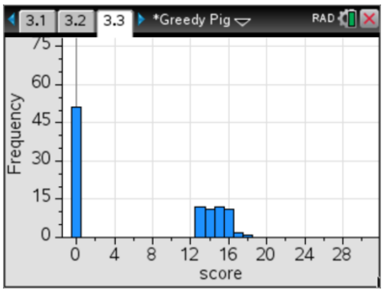


Question: 9

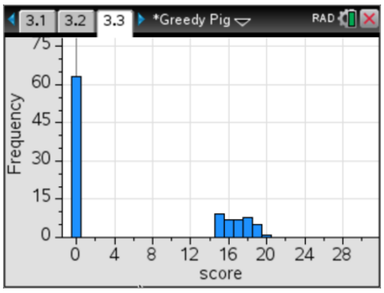
Discuss the data that you have obtained. You can include graphs and other statistical measures in your discussion.

Answer: The graphs are quite similar with the exception being that the results are less spread out. All maximum scores are within 5 points of the cut off score due to the nature of the strategy and no scores are recorded between 0 and the selected target score.

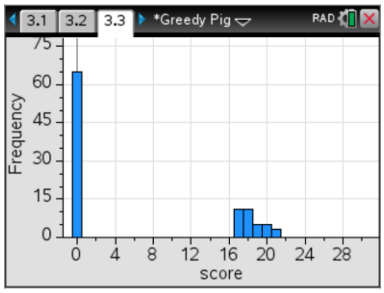
Score: 13



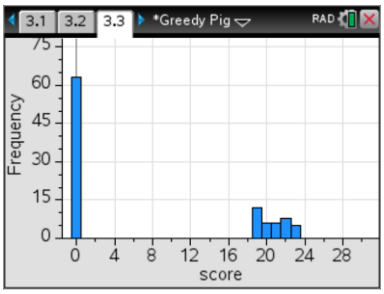
Score: 15



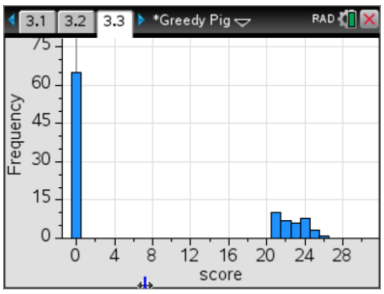
Score: 17



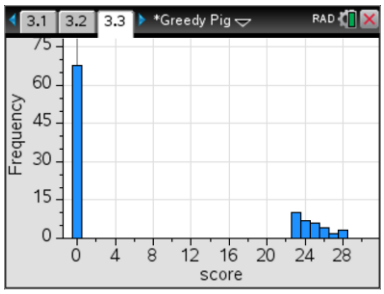
Score: 19



Score: 21



Score: 23



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Scoring Theory

Would you bank your score if you only had 3 points? What about 6 points? You may have already experienced that it is reasonably common for your score be returned to zero even for these relatively low totals, however, you would probably consider yourself unlucky. So the question is: "What score do you expect to get in any given game?"

The 'expected' score to be returned for any given roll can be calculated using the following:

$$E(x)=\frac{1}{6}\times 1-\frac{1}{6}\times s+\frac{1}{6}\times 3+\frac{1}{6}\times 4+\frac{1}{6}\times 5+\frac{1}{6}\times 6$$ s = Current score

To understand this formula, consider the following:

> "There is a 1/6 chance your score will increase by 1, (rolling a 1); there is a 1/6 chance your score will increase by 3 (rolling a 3) ... and a 1/6 chance your score will increase by 6; however there is a 1/6 chance your score will go back to zero, that is to lose (subtract) your current score (s)."

Question: 10

Determine the value of 's' for which your expected return would be zero and interpret this result.

Answer: Solving the linear equation produces: s = 19

Question: 11

Suppose your current score is 30 points. You are trying to decide whether or not to risk another roll. Determine the expected return for the next roll and interpret the outcome.

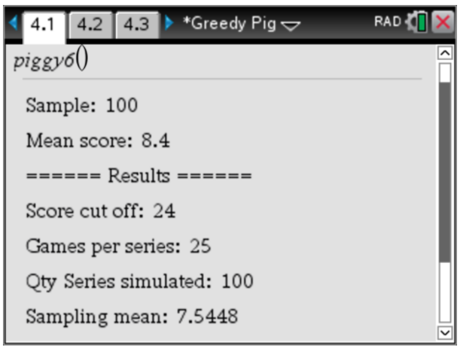
Answer: Students obtain the answer by substitution: -1.833...

Interpretation of the result: This value represents on average the amount by which the score is expected to change. There is a 5/6 chance the score could increase and a 1/6 chance it could decrease, however the amount by which it may decrease 'outweighs' the amount by which it might increase.

What to normally expect

Two strategies were explored in Questions 7 and 8. The data involved playing 100 games with scoring averages taken for each specific strategy. This was repeated three times for each strategy. Another way to explore the data is to collect averages (mean) from 100 series, with each series consisting of a specified quantity of games.

Navigate to page 4.1 of the Greedy Pigs document. This problem contains the last two Piggy programs. .



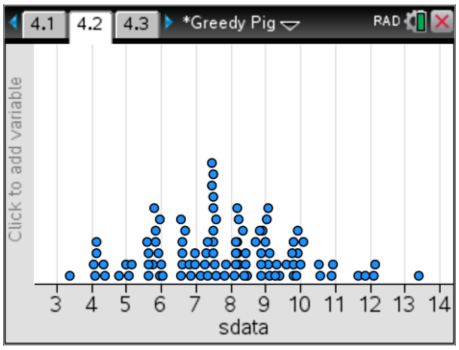
Piggy5 = Specify the number of rolls before banking the total, the number of games in a series and how many series you want to simulate.

Piggy6 = Specify the target score before banking the total, the number of games in a series and how many series you want to simulate.

If you select 100 games in a series and request 100 simulations, it will take a couple of minutes to complete the simulation as this is equivalent to 10,000 games!

Question: 12

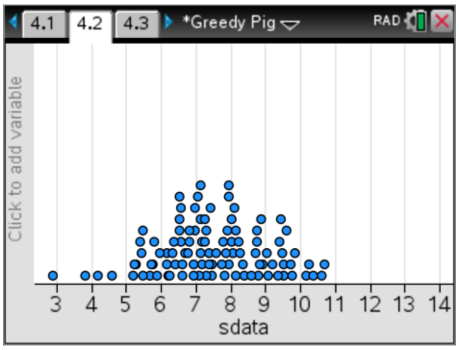
Consider targeting a score of 19 in each game. The aim is to obtain the highest average from a series of 25 games. Use the Piggy6 game to input these parameters and simulate 100 series. (samples)
Display and discuss the resulting graph for the distribution of series averages. (Sampling distribution)
Answer: The graph shows that there are very few games with a very low or high score, most of the games are located approximately symmetrically around the mean value. (7.79 in this sample).



Teacher Notes: This is a simple introduction to 'sampling' distributions. From previous questions student will have noticed that the sample means vary, this is a dynamic way of showing the variation between samples and a lead in to Year 11 / 12 mathematics.

Question: 13

Once again, consider targeting a score of 19 in each game. This time the aim is to obtain the highest average from a series of 50 games. Use the Piggy6 game to input these parameters and simulate 100 series. (samples)
Display and discuss the resulting graph for the distribution of series averages, specifically in comparison to the distribution contained in the previous question.
Answer: The graph is similar to the previous question, however the data is more closely packed around the mean. (Smaller standard deviation) The mean of the distribution is very similar (7.45).



Teacher Notes: This reinforces the notion that the mean is approximately the same for the sampling distribution however the large sample size produces a smaller variation.

Question: 14

How can we be more 'certain' about the mean for each strategy?
Answer: Increasing the quantity of games in each series decreases the spread of data providing more consistency in the estimate of the mean. (Concept of central limit theorem)

Question: 15

Working with a team of friends (to save time), complete the table below.
Quantity of games in each series: 50
Quantity of series to be simulated: 100

Target Score	13	15	17	19	21	23	25
Mean of distribution:							

Extension – Coding

To date the focus has been on obtaining the highest average for the series, but if we want to 'win', our strategy actually needs to reflect our current status. Consider the following two scenarios:

Footy Tipping Competition:

> *All season you've had a strategy of tipping the team that is higher on the ladder. This has put you close to the top of the competition. Suppose there are only two rounds remaining. Each round consists of 9 games; that is a total of 18 chances to gain extra points on your current position. The leader is 6 points clear. Most of the games seem to involve what is likely to be a clear result, due to relative ladder positions.*
>
> *Would you continue to use your strategy?*

One Day International – Cricket Match

> *The opposition has scored a total of 273 runs. Your team has 200 runs on the board at the end of the 40th over. This means your team has been averaging 5 runs per over, only slightly short of the original target of 5.46 runs per over. The problem now is that even if the original target of 5.46 runs is maintained the final score will be approximately 255 runs, close, but no win! Clearly more risks must be taken in order to win the match.*

The Piggy4 game is available for editing in Problem 5 as "mypiggy".

Step 1:

Work out how the program works with regards to individual games and the sampling of 100 games. Include a program listing and appropriate explanations next to each applicable line of code.

Step 2:

Determine and articulate a strategy adjustment to the targeted scoring system based on the current score and status of a 25 game series. (Don't share your ideas with anyone else! – Refer Step 4)

Step 3:

Convert your strategy into code to work with the program.

Step 4:

It's time to test your code. Play your 25 game series against an opponent and their 25 game series. Check the result at the end of each 25 game series to determine the winner. First person to win 20 series is the overall winner. Evaluate your strategy.

Teachers Notes: This is a coding task. The coding changes simply need to measure the current score average and see if it is below the expected value. If it is ... then increase the cut-off score slightly. Remember, the increased risk is likely to bring a lot more zero scores, however if the if you're chasing a 'win' the loosing margin is irrelevant.

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