Amazing Locus

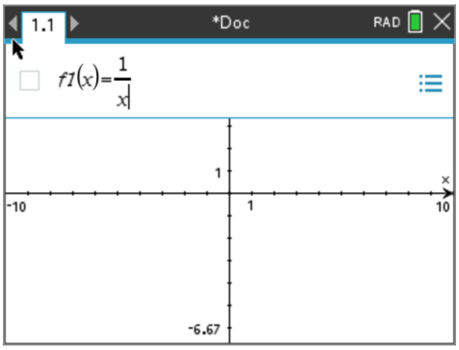
Teacher Notes and Answers

7 8 9 **10** **11** 12

TI-Nspire™ Investigation Student 50 min

" Find the equation to the locus formed by the intersection of the altitudes of a triangle with vertices on the function $y = \frac{1}{x}$ "

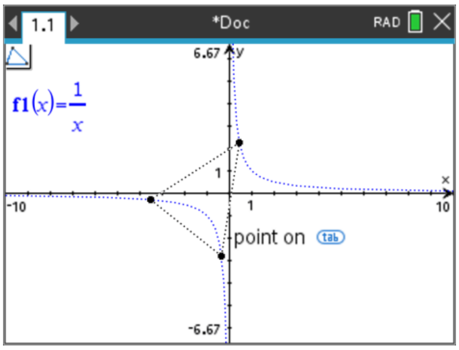
Exploring the Problem with Help from TI-Nspire



Start a new document and insert a Graphs Application.

We can break the problem statement into pieces, tackling the things that we know first. The problem talks about a "triangle with vertices on the function…", so before we draw the triangle we must attend to the function.

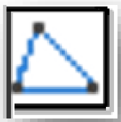
Graph the function: $y = \frac{1}{x}$



The next step in the problem is to draw the "triangle with vertices on the function …"

[Menu] > **Geometry** > **Shapes** > **Triangle**

The triangle tool appears on the top left corner of the screen. When this is active the mouse/cursor turns into a pencil. Move the pencil over the function until it says "point on", then click to place the vertex on the function.



Place all three vertices on the function, then press [esc] to release the triangle tool.

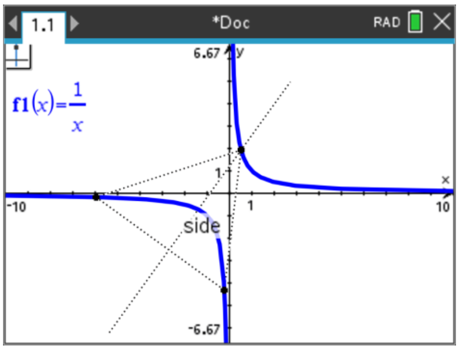
When several objects are close together on the screen it can be difficult to manually isolate a specific object. Think of the objects as layered. When the pointer is close to a collection of objects, press the **TAB** key to successively identify the nearby objects.

Authors: Peter Fox

Question: 1.

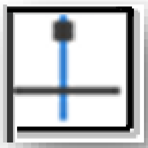
Where do the altitudes of the triangle appear to intersect?

Now we have a triangle with vertices on the function, we need to add in the altitudes. There are three altitudes. Each altitude passes through a vertex and is perpendicular to the opposite side.



[Menu] > **Geometry** > **Construction** > **Perpendicular**

With the perpendicular tool selected, once again the mouse/cursor changes to a pencil.



Click on a vertex, notice that the tool tip states “Point On”, select the vertex then the opposite side, the tool tip will display “Side”.

Repeat this process for all three vertices and their subsequent altitudes.

You can add a point of intersection between any two lines by using the “Point of Intersection” tool located in the Points and Lines options in the Geometry menu. On the CX II series, simply press “P”, select point and then navigate to the point of intersection.

The following naming system applies to the remaining questions. Vertices have been labelled for easy references and altitudes are named after the vertex they pass through.
For Questions 2 to 6 the coordinates for the triangle’s vertices are: A: (1, 1); B: (- ¼, - 4) and C: (- 4, - ¼) .

Question: 2.

i) Determine the gradient of line BC.

ii) Determine the gradient of altitude A.

iii) Determine the equation to altitude A.

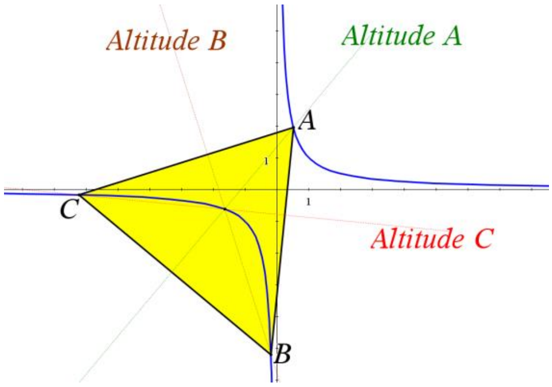


Figure 1 - Diagram not to scale

Question: 3.

Determine the equation for altitude B.

Question: 4.

Determine the equation for altitude C.

Question: 5.

Use simultaneous equations to determine the point of intersection between altitude A and B.

Question: 6.

Determine the point of intersection between altitude B and C and comment on your finding.

Question: 7.

Create your own set of coordinates for points A, B and C, making sure they are all on the function. Determine the point of intersection for your three altitudes.

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Question: 8.

Points A, B and C have coordinates: $\left(a, \frac{1}{a}\right)$, $\left(b, \frac{1}{b}\right)$ and $\left(c, \frac{1}{c}\right)$.

Determine the general equations for each altitude and the subsequent point of intersection.
Comment on your findings.

Question: 9.

Explore variations of this function to see if the locus continues to follow a distinct path:

i) $y = \dfrac{1}{x-h}$

ii) $y = \dfrac{1}{x} + k$

iii) $y = \dfrac{1}{ax}$

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