

# Probable Factors

Student Activity



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7 8 9 10 11 12



TI-Nspire CAS



Investigation



Student



120min

## Problem Statement:

What is the probability that a randomly generated quadratic can be factorised? This investigation looks at a significantly reduced set of quadratics using dice to generate the coefficients. Two dice are rolled; the numbers appearing uppermost are added together and form the  $x$  coefficient ( $b$ ) in the quadratic:  $y = x^2 + bx + c$ . Another two dice are rolled; the numbers appearing uppermost multiply together and form the constant ( $c$ ). The coefficient of  $x^2$  is 1. What is the probability this quadratic can be factorised:

- Over the rational number field?
- Over the real number field?

## Equipment

For this activity you will need:

- Four regular dice
- TI-nspire CAS

## Introduction – Setting up the simulation

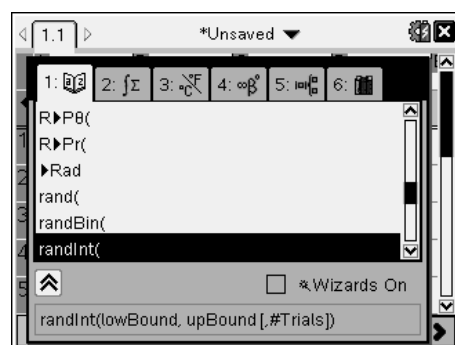
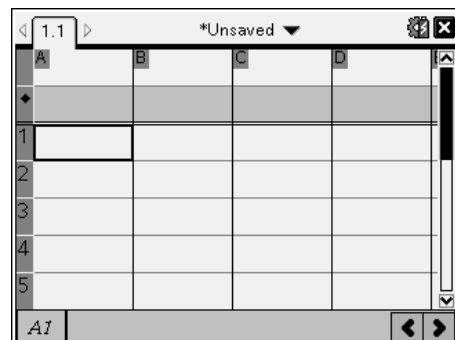
Start a new document and insert a spreadsheet. The spreadsheet will be used to generate the random integers (whole numbers) in place of the dice.

In cell A1 type the formula: =randInt(1,6)

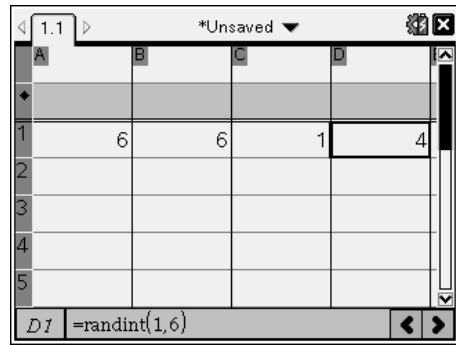
This produces a random integer between 1 and 6.

### Note:

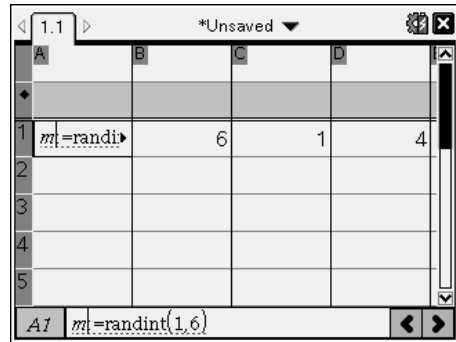
In the spreadsheet application the fastest way to access the **randInt()** command is through the catalogue.



The formula in cell A1 can be copied into cells B1, C1 and D1. Click on cell A1, press **CTRL + C** to copy, move to cell B1 and press **CTRL + V** to paste. Repeat this process for cells C1 and D1.



The simulated dice rolls need to be stored. Select cell A1 and press **CTRL + VAR** and store the dice roll in 'm'. Repeat this for cell B1, store as 'n'; cell C1 store as 'p' and cell D1 store as 'q'.

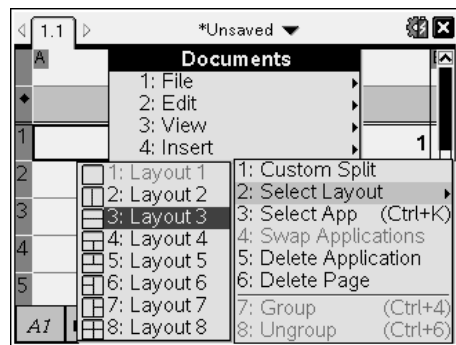


**Note:**

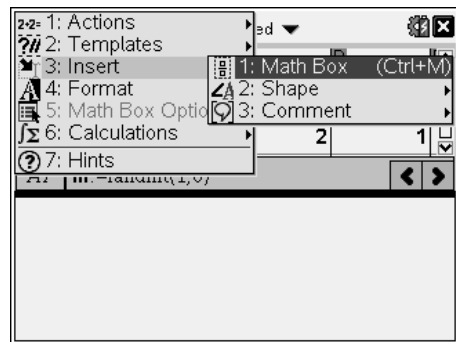
The simulated dice rolls become bold once they have been stored.

Press **CTRL + R** to simulate dice rolls and observe how each cell contains a new outcome each time CTRL + R is pressed.

The next step is to split the current page into two applications. Use the document key (or CTRL + Home on clickpad), select **Page Layout** followed by **Select Layout** and split the screen into two horizontal regions.



Use the mouse or **CTRL + Tab** to switch between applications. When the new region is active use the **menu** to insert a 'Notes' application.



In the new application area insert a Notes page.

On the notes page use the **menu** to **insert a Maths Box**.

In the Maths box type:  $x^2 + (m+n) \cdot x + p \cdot q$

**Note:**

A multiplication sign must be used between the parenthesis and the x, similarly between the p and q to ensure these operations are multiplied.

Notice the defined variables m, n, p and q appear bold. Furthermore the expression is automatically simplified as the output for the maths box. Can the expression be factorised?

In the second maths box type:  $\text{factor}(x^2 + (m+n) \cdot x + p \cdot q)$

The quadratic expression is random and based on the dice rolls simulated in the spreadsheet. The way the factor command has been set up in this situation, only rational factors will be displayed.

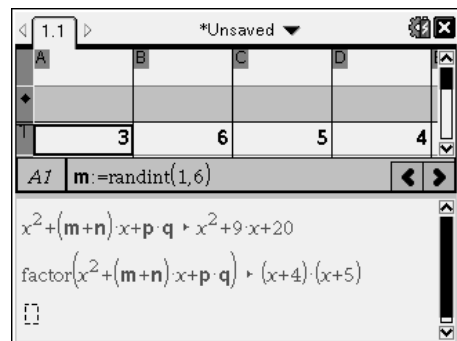
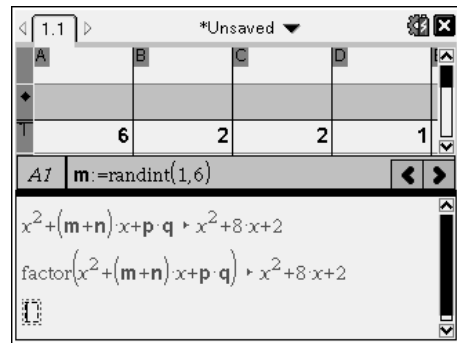
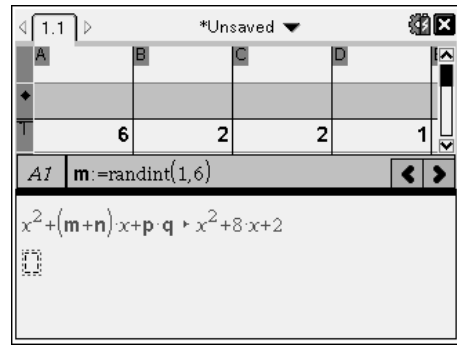
Switch control back to the spreadsheet by pressing **CTRL + Tab** or by using the mouse.

**Note:**

If cells A1 to D1 have disappeared, they may be hidden due to the reduced number of rows displayed on the screen.

With the focus on the spreadsheet, navigate to cell A1 and the simulated dice rolls will be displayed.

In the spreadsheet press **CTRL + R** to simulate another dice roll with subsequent expanded and potentially factorised quadratic trinomials.



## Probability through simulation

1. To get a sense of the probability of rolling a quadratic that will factorise over the rational number field, simulate 100 rolls of the dice. Watch the notes page and count how many of the quadratic equations factorise. Write your answer as a probability or percentage.

Your response to question one provides an indication of the actual answer. More simulations will produce a better approximation to the actual answer. The sample space for the dice sum (x coefficient) and dice product (constant) will help calculate the theoretical answer to this problem, a lattice diagram is one way to illustrate the sample space.

2. Complete the lattice diagram below for the sum of the two dice; some of the answers have been completed. The lattice diagram represents all the possible coefficients for  $x$ , in  $y = x^2 + bx + c$

Dice Sum	1	2	3	4	5	6
1	2					
2						
3		5				
4			7			
5						
6						

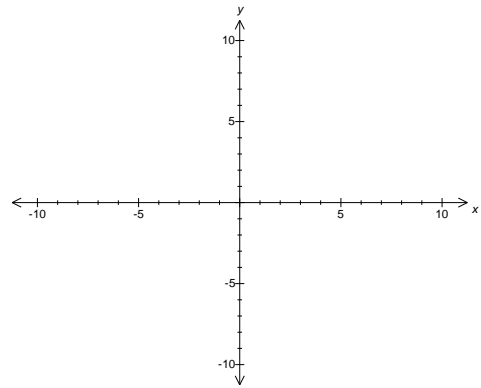
3. Complete the lattice diagram below for the product of the two dice; some of the answers have already been filled in. The lattice diagram represents all the possible constants in  $y = x^2 + bx + c$

Dice Prod.	1	2	3	4	5	6
1	1					
2						
3		6				
4			12			
5						
6						

The two lattice diagrams show the individual possibilities for the coefficient (b) and the constant (c). For each coefficient there are 36 permutations making a total of  $36 \times 36 = 1296$  possible equations to be factorised. Some of the equations are repeated, systematic exploration combined with some thinking and reflection will drastically reduce the time taken to explore which of these 1296 combinations factorise.

4. Suppose a [1] and a [1] are rolled on the first pair of dice to give an  $x$  coefficient of 2.  
 The equation would be:  $y = x^2 + 2x + c$  Two more dice must be rolled, the product of which form the constant (c). The lattice diagram for the product can be used to write down all the possible quadratics:  $y = x^2 + 2x + 1$ ,  $y = x^2 + 2x + 2 \dots y = x^2 + 2x + 36$
- a. Write down all the quadratics that can be formed.

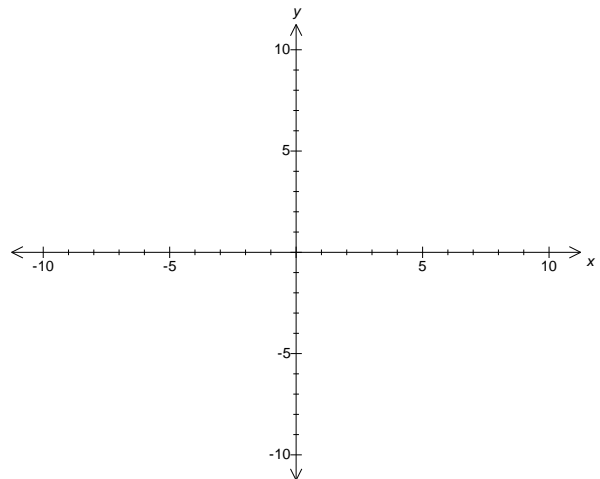
- b. Determine which quadratics can be factorised.
- c. Graph each of the equations indicating which ones factorised successfully over the rational number field.
- d. Use the lattice diagram for the dice products to determine how many of the quadratics of the form:  $y = x^2 + 2x + c$  will factorise.



5. Suppose a [5] and a [3] are rolled on the first pair of dice to give an x coefficient of 8. The equation would be:  $y = x^2 + 8x + c$ . Two more dice must be rolled, the product of which form the constant (c). The lattice diagram for the product can be used to write down all the possible quadratics:  $y = x^2 + 8x + 1$ ,  $y = x^2 + 8x + 2$  ...  $y = x^2 + 8x + 36$

- a. Write down all the quadratics that can be formed.
- b. Determine which quadratics can be factorised.
- c. Graph each of the equations indicating which ones factorised successfully over the rational number field.
- d. Consider both sets of graphs and observe the value for the constant where the family of graphs no longer cross the x axis.

- i. Comment on your findings.
- ii. Use this to predict or hypothesise the value of the constant (c) such that the family of graphs  $y = x^2 + 10x + c$  will no longer cross the x axis.



- iii. Check your hypothesis with your teacher and discuss how this can reduce the number of possible equations to be explored in this investigation.
- iv. Use the "dice product" lattice diagram to determine how many of the quadratic equations with an x coefficient of 8 will factorise?

- e. What other dice combinations are possible to produce an  $x$  coefficient of 8?
6. The modified lattice diagram below includes a space to record the number of quadratics that factorise for the corresponding  $x$  coefficient. Complete the lattice diagram with each of the associated results.

Dice Sum	1	2	3	4	5	6
1	2 1					
2						
3		5 2				
4			7 6			
5						
6						

7. From the completed lattice diagram determine the theoretical probability for the number of quadratics that can be factorised in this dice simulation and compare this result to the estimated probability using the simulations.

## Extension

What is the probability that a quadratic produced in the same way will factorise over the real number field?