Learning Exponentially

Student Activity

7 8 9 10 **11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

What happens to the graph of an exponential function when it is dilated, reflected or translated? What combinations of these transformations are homomorphic? In this investigation you will use TI-nspire technology to visually and numerically explore these transformations.

Tip

Q. Why can't Shetland ponies speak?

A. They're always a little *hoarse*.

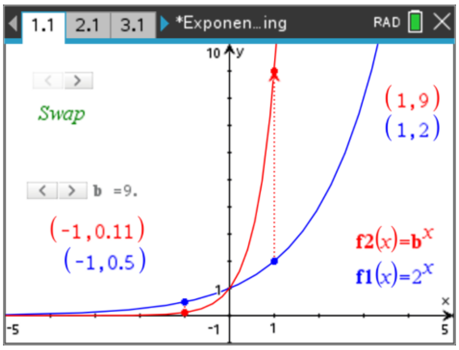
Jokes often rely on '**homophones**', words that spelt differently but pronounced the same. In mathematics, **homomorphic** expressions may look different but produce the same result.

Part A: Exploring $y = b^x$

Open the TI-Nspire file: Learning Exponentially.
Navigate to page 1.1
There are two functions in this graph application:

$$f_1(x) = 2^x \text{ and } f_2(x) = b^x$$

The purpose of Part A is to explore what happens to the points on the curve as the value of *b* is changed. Values for *b* range from 2 to 10.
Two points are identified specifically to help understand what is happening.
The movement of these points may be considered in the x or y direction.
Use the 'swap' slider to toggle the direction.



Question: 1

As the base (*b*) is changed from 2 through to 10:

a) Describe what happens to the points $(1,2)$ and $\left(-1,\frac{1}{2}\right)$ by consideration of the ordinate (y coordinate).

b) Which points, if any, are invariant? (Explain)

c) Describe what happens to the points $(1,2)$ and $\left(-1,\frac{1}{2}\right)$ by consideration of the abscissa (x coordinate).

d) Describe what happens to the *shape* of the graph as *b* is changed.

e) Identify the asymptote(s) for each of the graphs.

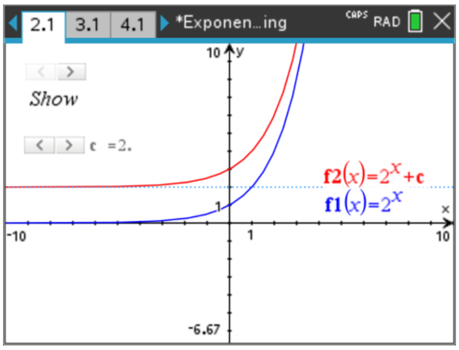
Author: Ireson & Fox

Part B: Exploring $y = 2^x + c$

Navigate to page 2.1
There are two functions in this graph application:

$$f_1(x) = 2^x \text{ and } f_2(x) = 2^x + c$$

The slider can be used to change the value of *c*.
Try values for *c*: ⁻5 through to 5.
The show / hide slider can be used to help visualise how individual points are moving.



Question: 2

As the value of *c* is changed from -5 through to 5:

a) Describe what happens to the *shape* and *location* of the graph.

b) Identify the asymptote(s) for each of the graphs.

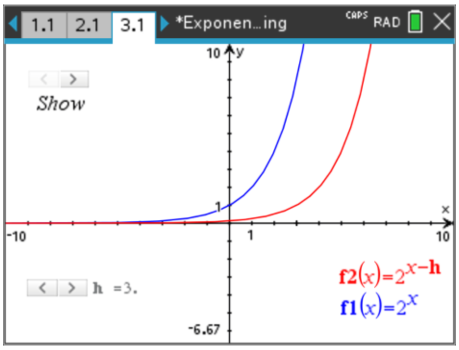
c) For what range of values for *c* does $f_2(x) = 0$ have a solution?

Part C: Exploring $y = 2^{x-h}$

Navigate to page 3.1
There are two functions in this graph application:

$$f_1(x) = 2^x \text{ and } f_2(x) = 2^{x-h}$$

The slider can be used to change the value of *h*.
Try values for *h*: ⁻5 through to 5.



Question: 3

As the values of *h* is changed from -5 through to 5:

a) Describe what happens to the *shape* and *location* of the graph.

b) Identify the asymptote(s) for each of the graphs.

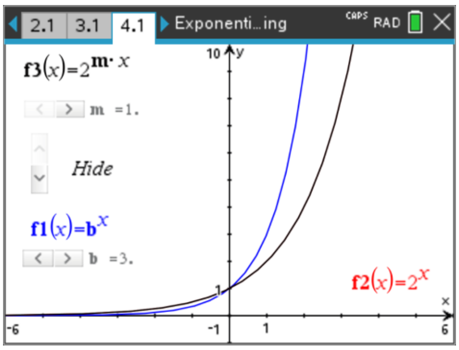
Part D: Exploring $y = 2^{mx}$

Navigate to page 4.1
There are three functions in this graph application:

$$f_1(x) = b^x \text{ , } f_2(x) = 2^x \text{ and } f_3(x) = 2^{mx}$$

The show/hide slider can be used to reveal the graph of $f_1(x)$.

Sliders can be used to change the values of *m* and *b* once the additional graph has been revealed.



Authors: Ireson & Fox

Question: 4

As the values of *m* is changed from 1 through to 4 in the graph of $f(x)=2^{mx}$:

a) Describe what happens to the *shape* and *location* of the graph.

b) Are there any invariant points on the graph of: $f(x)=2^{mx}$?

There is a show / hide button on the Graph [Page 4.1]. Use this to reveal another graph that has similar form, one that has been explored previously.

Question: 5

Explore values of *m* and *b* for the graphs of: $f(x)=2^{mx}$ and $f(x)=b^{x}$, determine values for which the graphs are homomorphic (same) and explain the outcome.

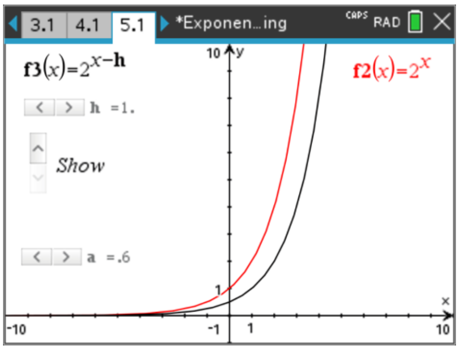
Part E: Exploring $y=a\cdot 2^{x}$

Navigate to page 5.1
There are three functions in this graph application:

$$f_1(x)=a\cdot 2^{x},\ f_2(x)=2^{x} \text{ and } f_3(x)=2^{x-h}$$

The show/hide slider can be used to reveal the graph of $f_1(x)$.

Sliders can be used to change the values of *a* and *h* once the additional graph has been revealed.



Question: 6

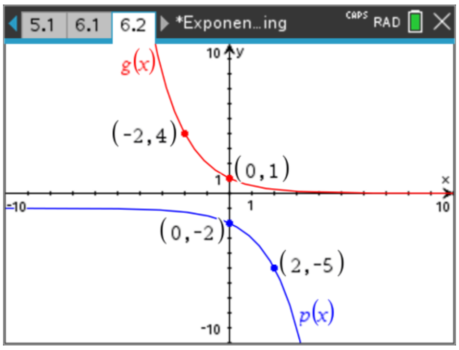
Explore values of *a* and *h* for the graphs of: $f(x)=2^{x-h}$ and $f(x)=a\cdot 2^{x}$, determine values for which the graphs are homomorphic (same), explain the outcome and identify limitations for the comparison..

Question: 7

Compare the transformations of $f(x)=2^{x-h}+c$ with those for $g(x)=(x-h)^2+c$.

Part F: Combining Transformations

Navigate to page 6.1
This problem contains a blank graph application for you to use with the aim of matching the two graphs shown opposite. Each graph has been created using just one of the transformation parameters explored so far. The values for the parameters however are outside those explored.



Question: 8

The graph of $g(x)$ is of the form: $g(x)=a\cdot 4^{mx}+c$. Identify possible values for the parameters such that the function passes through (-2, 4) and (0, 1) and has an asymptote at $y=0$.

Question: 9

The graph of $p(x)$ is of the form: $p(x)=a\cdot 8^{mx}+c$. Identify values for the parameters such that the function passes through (2, -5) and (0, -2) and an asymptote at y = -1.

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