Repeating Decimals and Fractions

Math Objectives

- Students will explore and predict patterns found in repeating decimals for select fractions.
- Students will determine the repeating digit (or group of digits) for a repeating decimal.
- Students will find repeating digits and use patterns in repeating decimals to determine equivalent fractions.
- Students will understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion that repeats eventually into a rational number (CCSS).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

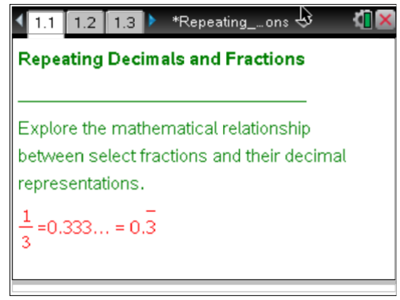
- decimal representation
- repeating decimal
- repeating digits

About the Lesson

- This lesson involves students exploring patterns in repeating decimals that represent fractions with denominators 9, 99, and 999.
- As a result, students will:
  - Find the repeating decimal representation for select fractions.
  - Find fractions given select repeating decimal representations.

TI-Nspire™ Navigator™ System

- Send and collect a file.
- Use Live Presenter to monitor student work and generate class discussions.
- Use Screen Capture to monitor student work and generate class discussions.
- Use Quick Poll to assess students' understanding and generate class discussions.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- In order move the slider, place a cursor over it and press ctrl to grab the slider.

Lesson Files:

Student Activity

Repeating_Decimals_and_Fractions_Student.pdf

Repeating_Decimals_and_Fractions_Student.doc

TI-Nspire document

Repeating_Decimals_and_Fractions.tns

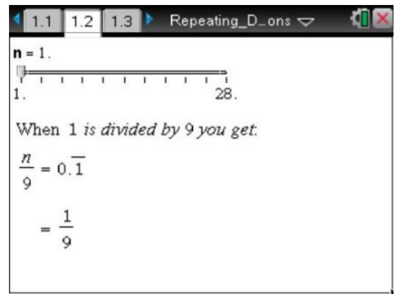
Repeating_Decimals_and_Fractions_Assessment.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Teacher Tip: If your students have never used the “bar” notation for repeating decimals, explain the meaning of the “bar” before the students begin their exploration. The name for the bar is a **vinculum**. The repeating digit (or group of digits) is called **repetend**, so the bar is always placed over the repetend.

Move to page 1.2.



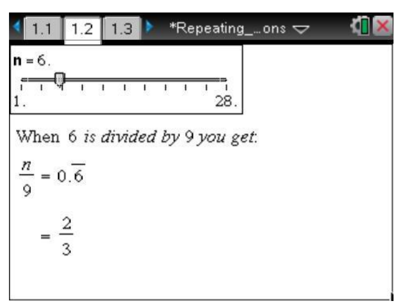
1. First, explore fractions with a denominator of 9. Use the slider to change the numerator of the fraction, and observe the changes in the decimal representation of the fraction. Record your observations in the table below.

   <u>Answer:</u>

Fraction	Decimal representation
$\frac{1}{9}$	$0.\overline{1}$
$\frac{2}{9}$	$0.\overline{2}$
$\frac{4}{9}$	$0.\overline{4}$

2. Describe any patterns that you observed.

   <u>Answer:</u> When you have 9 for the denominator, the digit in the numerator becomes the repeating digit in the decimal representation of the fraction.



3. Based on your observations predict the repeating decimals for the fractions given in the table below. Then, use the slider to check your predictions, and record the actual repeating decimal in the table.

Answer:

Fraction	Predicted repeating decimal	Actual repeating decimal
$\frac{5}{9}$	0.555…	$0.\overline{5}$
$\frac{6}{9}$	0.666…	$0.\overline{6}$
$\frac{7}{9}$	0.777…	$0.\overline{7}$
$\frac{8}{9}$	0.888…	$0.\overline{6}$

Teacher Tip: The TI-Nspire will always display the fraction in reduced form. Discuss equivalent fractions with students, for example, $\frac{6}{9}=\frac{2}{3}$. That happens whenever the numerator and denominator have common factors.

Tech Tip: Students can find the decimal representations for fractions using Scratchpad by pressing [Scratchpad]. The TI-Nspire uses division to compute the decimal values. These decimals are displayed as rounded decimals on the calculator screen. In order to return to the document, close the Scratchpad by clicking ☒ in the top right corner or just press [esc].

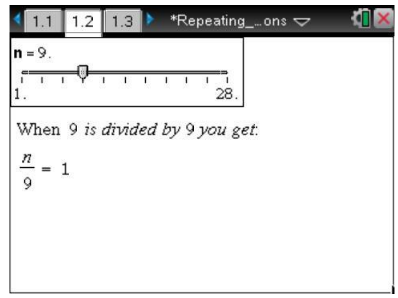
TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note at the end of this lesson.

4. What is the repeating digit in the decimal representation of the fraction $\frac{n}{9}$ when *n* ranges from 1 to 8?

Answer: The repeating digit in the decimal representation of the fraction $\frac{n}{9}$ is *n*, e.g.

$$\frac{n}{9}=0.nnn....=0.\overline{n}.$$

Teacher Tip: Help students analyze patterns in order to establish the relationships between the numerator of each fraction and repeating digit in the repeating decimal for each fraction. This pattern is often very easy for students to find. Encourage students to explain the pattern. This might be a good time to demonstrate a division problem like 7 ÷ 9 as shown in the Repeating Decimals Appendix document to illustrate how the long division algorithm reveals the same remainder causing the next digit in the quotient to be the same. Then ask students to compute 8 ÷ 9 on their own in order to find the ever-repeating remainder and the digit "8" that repeats. Encourage your students to explain why this algorithm yields repeating digit.

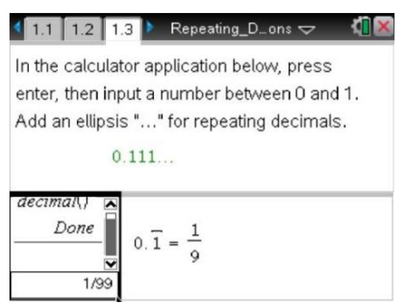


5. Move the slider to find three fractions with decimal representation that do not have repeating digits. What do you notice about these fractions? Why does the decimal representation have no repeating digits?

Answer: $\frac{9}{9}$, $\frac{18}{9}$ and $\frac{27}{9}$. These fractions have numerators that are multiples of 9. The decimal representations for these fractions do not have repeating digits because the denominator 9 divides each of the numerators exactly.

Teacher Tip: You might need to remind your students that all rational numbers can be expressed as fractions or decimals with a finite number of digits or infinitely repeating digits. Numbers like 2.34 and 0.265 are rational and are sometimes called terminating decimals because the number of digits after the decimal point is finite. Numbers that have an infinite number of digits in the decimal expansion and have no repeating digits are called irrational. For example, the numbers π and $\sqrt{2}$ are irrational numbers because they have an infinite number of digits after the decimal point but have no repeating digits. Johann Lambert proved that π is irrational in the 18th century.

Move to page 1.3.



Next, explore repeating decimals, and find the fractions for those decimals. Notice that Page 1.3 is divided into three regions:

- The top work area provides directions for the Calculator App
- The Calculator App is in the lower left work area of the screen.
- The lower right work area contains the equivalent decimal and fraction for the number you will enter.

To use the Calculator App, that work area must be active. It is active when there is a dark border around it. If there is no dark border, press ctrl tab until the border becomes dark.

Follow the instructions below for entering decimals into the program:

- The decimal values entered must be between 0 and 1.
- The decimal must begin with "0."
- The decimal must not contain more than ten digits after the decimal point.
- The repeating digit (or group of digits) should be included at least three times followed by the ellipsis "…" to indicate the number is a repeating decimal.

Tech Tip: When students first move to page 1.3 the Calculator App should be already active and shown in a bold frame. Use Screen Capture to verify that all students are in a correct place on this page. If not, instruct students to move to the Calculator app. Use computer software or Live Presenter to model this process. Only then should students proceed with the exploration.

6. In the table below, predict fractions for the given decimals with one repeating digit.
   Answer:

Decimal with one repeating digit	Predicted fraction	Actual fraction
0.22222…	$\frac{2}{9}$	$\frac{2}{9}$
0.44444…	$\frac{4}{9}$	$\frac{4}{9}$
0.66666…	$\frac{6}{9}$	$\frac{2}{3}$
0.99999…	$\frac{9}{9}$	1

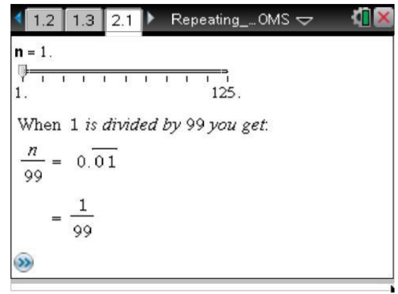
7. Press enter to activate the program in the Calculator work area. In the pop-up window, input each decimal with one repeating digit, one at a time. Press OK. Record actual fractions in the table above.

8. Did any repeating decimal with one repeating digit result in a fraction with a denominator not equal to 9? Why?

   Answer: $\frac{2}{3}$ and 1. The fraction $\frac{2}{3}$ is equivalent to $\frac{6}{9}$. The fraction $\frac{9}{9}$ is equivalent to 1.

Teacher Tip: Students might make an entry error on Page 1.3. If that happens, have them press ctrl esc repeatedly to undo the error(s).

Move to page 2.1.



9. Now, let's explore fractions with denominator 99. Use the slider to change the numerator, and observe the changes in the decimal representation of the fraction. Record your observations in the table below.

Answer:

Fraction	Decimal representation
$\frac{1}{99}$	$0.\overline{01}$
$\frac{5}{99}$	$0.\overline{05}$
$\frac{10}{99}$	$0.\overline{10}$
$\frac{27}{99}$	$0.\overline{27}$

10. Based on your observations predict the repeating decimals for the fractions given in the table below. Then, use the slider to check your predictions, and record actual repeating decimals in the table.

Answer:

Fraction	Predicted repeating decimal	Actual repeating decimal
$\frac{22}{99}$	0.222…	$0.\overline{2}$
$\frac{23}{99}$	0.232323…	$0.\overline{23}$
$\frac{66}{99}$	0.666…	$0.\overline{6}$
$\frac{98}{99}$	0.989898…	$0.\overline{98}$

11. Use the long division algorithm to compute 23 ÷ 99. What do you notice about the remainders appearing in each step of division? Why do you get the two repeating digits when dividing 23 by 99?

Answer: The long division algorithm for 23 ÷ 99 reveals the remainders 23 and 32 appear over and over. The digits 2 and 3 in the quotient appear over and over because of the repetition of the remainders 23 and 32.

$$\begin{array}{l} \quad\;\; 0.23 \\ 99\overline{)23.000} \\ \quad\; \underline{0} \\ \quad\; \mathbf{23}0 \\ \quad\; \underline{198} \\ \qquad \mathbf{32}0 \\ \qquad \underline{297} \\ \qquad\;\; \mathbf{23}0 \end{array}$$

12. Move the slider to find at least three fractions with decimal representations that have one repeating digit. What do you notice about these fractions?

    Answer: Fractions like $\frac{11}{99}$, $\frac{22}{99}$, and $\frac{33}{99}$, with numerators that are multiples of 11 all have decimal representations with one repeating digit, since they reduce to the fractions with denominator 9.

13. Can you find a fraction that does not result in repeating decimal? If so, explain why there are no repeating digits.

    Answer: The fraction $\frac{99}{99}$, has no repeating digits since 99÷99 = 1.

Move back to page 1.3

14. Using the program in Calculator App, find fractions for the decimals given in the table. Record your findings in the table.

    Answer:

Decimal with two repeating digits	Fraction
0.292929…	$\frac{29}{99}$
0.343434…	$\frac{34}{99}$
0.717171…	$\frac{71}{99}$

15. What patterns do you observe for the decimals with two repeating digits?

    Answer: The decimals with two repeating digits can be written as fractions with the numerator equal to the repeating digits and with denominator 99.

16. Record several decimals with two repeating digits in the table below, and predict the equivalent fractions based on your earlier observations. Then check your predictions on Page 1.3.

    Answer: Student choices of fractions and predictions will vary.

17. What is a common characteristic of fractions with decimal representation containing two repeating digits?

    Answer: These fractions are equivalent to fractions with denominators equal to 99.

18. Now predict fractions for the given decimals with three repeating digits. Check to see if your predictions are correct. What do you notice about the decimals that have three repeating digits?

    Answer: the fractions have denominator 999 and numerator equal to the number represented by 3 repeating digits.

Decimal with three repeating digits.	Predicted fraction	Actual Fraction
0.235235235…	$\frac{235}{999}$	$\frac{235}{999}$
0.707707707…	$\frac{707}{999}$	$\frac{707}{999}$
0.997997997…	$\frac{997}{999}$	$\frac{997}{999}$

19. What is a common characteristic of fractions with decimal representation containing three repeating digits?

    Answer: Each of these fractions has 999 as the denominator.

Tech Tip: If students want to access the Scratchpad Calculator, they need to press [scratchpad] when they are working in a document. When they want to return, the need to press [esc].

20. Explain how division might result in a decimal with a three repeating digits.

Answer: When you divide by 999 using the long division algorithm, there are three distinct 3-digit remainders that have same three digits in a repeating sequence, for example, 123, 231, 312, 123, etc.

21. Record your predictions for the decimals given in the table below. Check to see if your predictions are correct. What do you notice about the fractions for these repeating decimals? Explain why the denominator is not equal to 999.

Answer: The fractions are displayed as reduced fractions since the number represented by repeating digits have common factors with 999. For instance, the common factor of 123 and 999 is 3; common factor of 81 and 999 is 27.

Decimal	Predicted fraction	Actual fraction
0.123123123…	$\frac{123}{999}$	$\frac{41}{333}$
0.081081081…	$\frac{81}{999}$	$\frac{9}{111}$

22. Can you find fractions with the denominator 999 that have only one repeating digit? What do you notice about these fractions?

Answer: Example of fractions: $\frac{333}{999};\frac{555}{999}$. These fractions have numerators that are multiples of 111, and thus they are reduced to fractions with denominator equal to 9.

Teacher Tip: Encourage your students to use pattern analysis to establish the relationships between fractions and repeating decimals. The following questions could be used in the discussion:

- When you have a decimal with one repeating digit, what is the denominator for the equivalent fractions? They should know the denominator is a factor of 9.
- When you have a decimal with two repeating digits, what is the denominator for the equivalent fractions? They should know the denominator is a factor of 99.
- When you have a decimal with three repeating digits, what is the denominator for the equivalent fractions? They should know the denominator is a factor of 999.

23. Input decimals given below into the program on Page 1.3 and record the equivalent fractions. What do you notice about the denominators of these fractions? How can you predict the numerators?

Answer: The fractions displayed have 90 as the denominator. The decimal has one non-repeating digit, and the denominator has one zero. The numerators can be predicted by subtracting the digit that does not repeat from the first two digits after the decimal. For instance, 14 – 1 yields 13, the numerator of the decimal 0.1444… while 21 – 2 gives 19, the numerator for the decimal 0.2111… and the numerator for 0.4777… is 43 (47 – 4).

Decimal	Actual fraction
0.1444…	$\frac{19}{90}$
0.2111…	$\frac{13}{90}$
0.4777…	$\frac{43}{90}$

24. Based on your observations, record predicted fractions for the decimals given in the table below. Use the program on Page 1.3 to see if your predictions are correct. Explain the actual results.

Answer: 0.1333… = 12/90 but is displayed as the reduced fraction 2/15 because of the common factor of 6 for the values 12 and 90. 0.5111… = 46/90 but is displayed as 23/45, while 0.3363636… = 33/90 displayed as 11/30.

Decimal	Predicted fraction	Actual fraction
0.1333…	$\frac{12}{90}$	$\frac{2}{15}$
0.5111…	$\frac{46}{90}$	$\frac{23}{45}$
0.3363636…	$\frac{33}{90}$	$\frac{11}{30}$

25. Now record your predictions for a different set of decimals given below. Check to see if your predictions are correct. Explain how you can predict the numerator and denominators.

Answer: The decimal has two non-repeating digits, so denominator should have two zeros. Thus, denominator should be 900. The numerator can then be found by subtraction: 132 – 13 = 119 for 0.13222…; 81 – 8 = 73 for 0.08111…; 974 – 97 = 877 for 0.97444….

Decimal	Predicted fraction	Actual fraction
0.13222…	$\frac{119}{900}$	$\frac{119}{900}$
0.08111…	$\frac{73}{900}$	$\frac{73}{900}$
0.97444…	$\frac{877}{900}$	$\frac{877}{900}$

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to determine the repeating decimal for a fraction with varying numerators and denominators equal to 9, 99, and 999.
- How to determine the fraction for a repeating decimal with one, two, or three repeating digits.
- How to determine the repeating decimal for a fraction with varying numerators and a denominator equal to 90 and 900.
- How to verify decimal expansion of fractions by division.

Assessment

Use provided Repeating_Decimals_and_Fractions_Assessment.tns document to assess students understanding of the major concepts of this lesson. You can send the document as a file or use Quick Poll to send one question at a time.

TI-Nspire Navigator

Note

Name of Feature: Quick Poll

Use Quick Poll throughout the lesson to have students share the fractions or decimals they have found.