Power Function Inverses

TEACHER NOTES

MATH NSPIRED

Math Objectives

- For power functions of the form **f**(x)=x^n, where n is a positive integer and the domain is all real numbers, students will be able to identify which functions are invertible (odd powers) and be able to graph the inverse function using reflection symmetry across the line $y = x$.
- Students will be able to identify a suitable restricted domain for an even power function to have an inverse, and be able to graph the inverse function using reflection symmetry across the line $y = x$.
- Students will use appropriate tools strategically. (CCSS Mathematical Practice)

Vocabulary

- inverse
- domain
- one to one functions
- reflection
- horizontal/vertical line test

About the Lesson

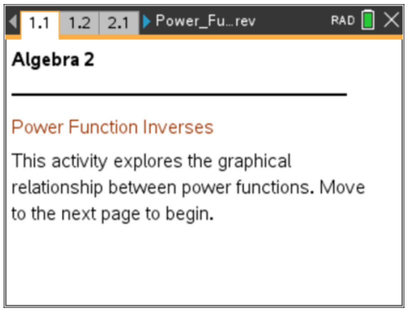
- This lesson involves students examining the graphs of power functions with even and odd integer powers. They will be looking for conditions under which the functions have inverses.
- As a result, students will:
  - Be able to recognize graphs of inverses as reflections over the line $y = x$.
  - Be able to find points on the graph of an inverse function by exchanging the x and y coordinates for points on the original function.
  - Be able to state when power functions have inverses (i.e., odd power functions with domains for all real x, and even power functions with the domain restricted to values $x \geq 0$.)

TI-Nspire™ Navigator™

- Use Class Capture and/or Live Presenter to monitor student progress and demonstrate the correct procedures.
- Quick Poll may be used to assess students' understanding of the concepts throughout or after the lesson.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Algebra 2

Power Function Inverses

This activity explores the graphical relationship between power functions. Move to the next page to begin.

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Power_Function_Inverses_Student.pdf
- Power_Function_Inverses_Student.doc

TI-Nspire document

- Power_Function_Inverses.tns

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: ***File Transfer***

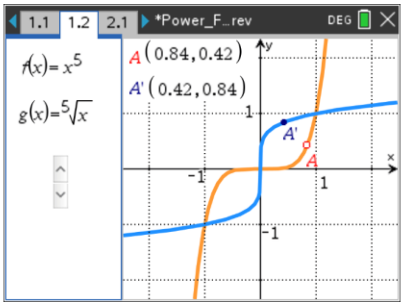
See Note 1 at the end of this lesson.

Tech Tip: On page 1.2 have students click the slider before moving the point because *A* and *A'* overlap initially. Be sure that students get the open hand icon (✋) before grabbing the point *A*.

Tech Tip: Consider showing students an alternative method for controlling the slider. When a page has sliders, click tab until the appropriate slider is “boxed” and press enter to select it. Then use the arrow keys to change the value of the slider variable (▲ or ▶ increase the value of the variable; ▼ or ◀ decreases the value of the variable). If the slider is already boxed, then press enter to select it.

Tech Tip: Tap on the arrows to change the values of the slider.

Move to page 1.2.



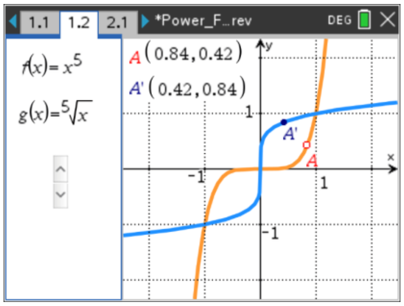
1. As you use on the slider, the graphs of $f(x) = x^p$ and $g(x) = \sqrt[p]{x}$ are displayed on the page for odd values of *p* from 1 to 15. These functions are inverses of one another. What geometric relationship exists between the two graphs?

   <u>Answer:</u> The graphs are reflections over the line *y* = *x*. This supports the fact that they are inverses when *p* is odd, because the graphs of inverse functions are reflections over the line *y* = *x.*

TI-Nspire Navigator Opportunity: ***Class Capture*** **or** ***Live Presenter***

See Note 2 at the end of this lesson.

Teacher Tip: The line *y* = *x* is called the identity function (or the identity line). Discuss with students reasons why the line *y* = *x* is called the identity function. (It is called the identity function because the values of *y* are identical to the values of *x*.) Be sure to remind students that inverse functions are reflections over the identity function.



2. A trace point, *A*, is placed on the graph of $f(x) = x^p$ and is represented by the open circle. As you drag point *A* along the function, the related point *A′* on the graph of $g(x)=\sqrt[p]{x}$ is updated as well. What relationship exists between the coordinates of *A* and *A′*?

Answer: As the point *A* is moved along its entire graph, the point *A′* also moves along its entire graph. The relationship between the coordinates of *A* and *A*' is that the *x*- and *y*-coordinates are exchanged, which is further evidence of the inverse relationship.

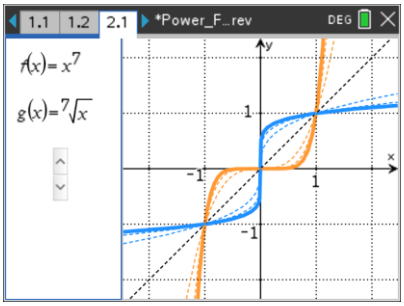
Teacher Tip: Some of the screens can take a few seconds to re-draw when using the handheld. The same actions go much faster on the computer software. Allow students time to explore the document.

3 Find (*f*)(*g*)(*x*) and (*g*)(*f*)(*x*). What is the result?

Answer: In both cases the result is *x*. This is another way to determine inverse functions.

Teacher Tip: There are three properties of inverse functions: (1) They are reflections over $y = x$. (2) The *x*- and *y*-coordinates are exchanged. (3) The composition of both functions in either order results in *x*. This would be a good opportunity to remind your students about all three.

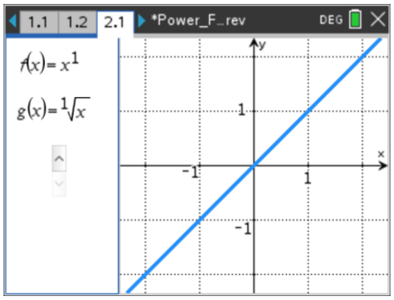
Move to page 2.1.



4. As the slider is pressed, a “trail” of graphs remains as *p* changes in odd values.
   a. The points (1, 1), (0, 0), and (–1, –1) are common to all of the graphs on this page. Using what you learned in question 2, explain why these points are common to all power functions of odd degree and their inverses.

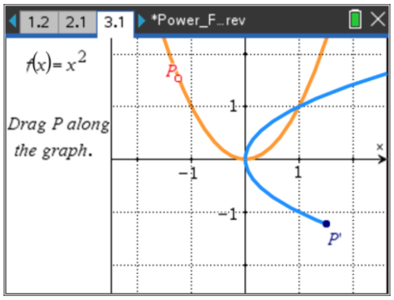
Answer: These points are common to all the graphs because they are the only three points on a power function with odd degree that produce the same point when you exchange their *x*- and *y*-coordinates. In other words, all three points are located on the identity line over which the graph reflects to produce the inverse.

b. What do you see when $p = 1$? Why does this happen?



Answer: When the value of p is 1 the function is $y = x$. This is the line of reflection (i.e., the identity line). It is also its own inverse.

Move to page 3.1.



Drag P along the graph.

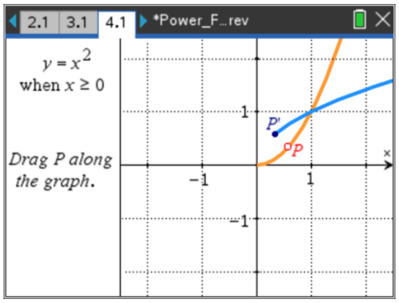
5. The graph of $f(x) = x^2$ is displayed on this page. A trace point, P, has been added. P', the point reflected over $y = x$, is also displayed. Drag the point P and watch the path of P'. Describe what you see after you drag point P over the entire graph of $f(x)$.

Sample answer: Student answers may vary, but should describe the picture to the right. This page is designed to help students realize that the reflection of a parabola opening upward is a parabola opening to the right.

6. Inverse functions must retain the properties of functions. Why does the resulting graph of the reflection of $f(x) = x^2$ over the line $y = x$ fail to meet this condition?

Answer: The graph followed by the path of P' is not a function because it fails the vertical line test, which means that there is more than one y-value for a given x-value.

Move to page 4.1.



Drag P along the graph.

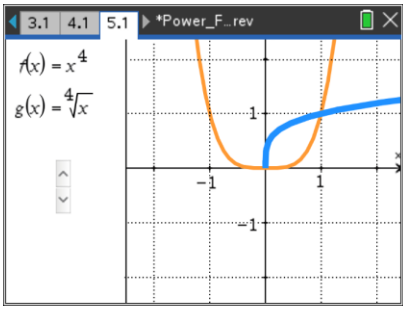
7. The graph of $f(x) = x^2$ is displayed on this page but this time only when $x \geq 0$. Again, the trace point P is displayed, as well as P', its reflection over $y = x$. Drag the point P and watch the path of P'. How does restricting the domain of $f(x)$ to $x \geq 0$ allow the function to have an inverse?

Answer: The reflection of the $f(x) = x^2$ restricted to $x \geq 0$ is a function because it passes the vertical line test.

8. The domain restriction $x \geq 0$ allowed the graph in question 7 to have an inverse. List another possible domain restriction for $f(x)$ that will allow there to be an inverse.

   Sample answer: Student answers may vary, but all answers should have the reflection pass the vertical line test. Sample answers could be $x \geq 2$, $x \leq 0$, $x \geq 5$, $x \leq -4$, etc.

Move to page 5.1.



$f(x) = x^4$

$g(x) = \sqrt[4]{x}$

9. As you press on the slider, the graphs of $f(x) = x^p$ and $g(x) = \sqrt[p]{x}$ are displayed for even values of p from 2 to 8. The geometric relationship observed for odd values of p no longer holds. Why does this geometric relationship fail to happen for even values of p?

   Answer: This fails because radical functions are customarily defined as the principal (or positive) root. For these functions to be inverses, the radical would have to return both the positive and negative root, which is not possible. When a power function has an even exponent, it is not a one-to-one function (so it does not pass the horizontal line test). Therefore, it does not have an inverse.

10. Based on the graphs on this page, which part of a power function with even degree is a reflection of a radical function with the same index?

    Answer: If the domain is restricted to positive numbers, an even degree power function will be the reflection of a radical function of the same index.

11. How can you tell visually from any graph of a function whether it will have an inverse or not? Why might this be useful?

    Answer: If the graph of a function passes the horizontal line test, it will have an inverse. This means that if a horizontal line passes over the graph and intersects it at only one point along the entire graph, then the function will have an inverse. If the graph does not meet this condition, then a restriction on the domain can allow a portion of it to be invertible.

12. Jorge claims that $f(x) = x^2$ and $g(x) = \sqrt{x}$ are inverses because squaring and square roots are "opposite operations." What has Jorge not considered in his conclusion?

Answer: Jorge has not considered the domains of the functions. If the domain of the quadratic was restricted to non-negative values, he would be correct.

Teacher Tip: Student answers may vary because some may struggle to visualize diagonal reflections. This document may help students who have difficulty. Having students position the calculator so that the line of reflection is vertical may help kinesthetic learners visualize it better.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to find an inverse of a power function with an odd integer exponent as a reflection over the line $y = x$.
- How to restrict the domain of a power function with an even integer exponent to allow it to have an inverse.

TI-Nspire Navigator

Note 1

Entire Document, *File Transfer:* Use *File Transfer* to efficiently send the TI-Nspire document file to students. Using TI-Navigator will allow students to receive the file without having to leave their seats or use extra cables.

Note 2

Entire Document, *Class Capture* or *Live Presenter:* If students experience difficulty with the syntax of any question, use *Class Capture* or *Live Presenter* with TI Navigator to demonstrate the correct syntax for the class.

Note 3

End of Lesson, *Quick Poll:* A *Quick Poll* can be given at the conclusion of the lesson. You can save the results and use the Review to show then at the start of the next class to discuss possible misunderstandings students may have.

The following are some sample questions you can use:

1. Which one of the following functions will have an inverse?
   a. $y = x^{-2}$
   [✓ b. $y = x^{73}$]
   c. $y = |x|$
   d. $y = x^{100}$

2. What could be a restricted domain of $y = (x - 2)^2$ for it to have an inverse?
   a. $x \geq 0$
   b. $x \leq 4$
   [✓ c. $x \geq 2$]
   d. $x \geq -2$

3. True or False: If a function fails the horizontal line test, it will never have an inverse that is a function.

   Answer: True