Properties of Parallelograms

About the Lesson

In this activity, students will explore the various properties of parallelograms using the dynamic properties of the Cabri™ Jr App. As an extension, students can also explore necessary and sufficient conditions that guarantee that a quadrilateral is a parallelogram. As a result, students will:

- Use technology to discover the properties of parallelograms.

Vocabulary

- congruent
- consecutive sides
- consecutive angles

Teacher Preparation and Notes

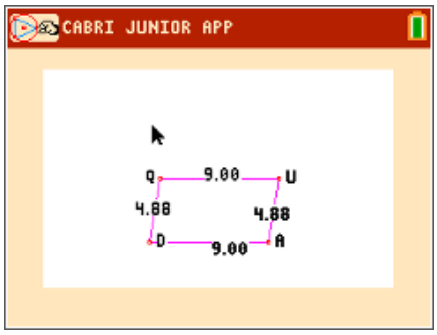
- This activity was written to be explored with the Cabri™ Jr. application on the TI-84 Plus family of graphing calculators.
- To grab a point in Cabri™ Jr., hover the cursor over the point and press alpha. To release, press alpha or enter. To move a point after grabbing it, use the arrow keys

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Properties_of_Parallelograms_Student.doc
- Properties_of_Parallelograms_Student.pdf
- PAR1.8xv
- PAR2.8xv
- PAR2.8xv

Tech Tip: Before beginning the activity, the Cabri™ Jr files PAR1.8xv, PAR2.8xv, and PAR3.8XV need to be transferred to the students' calculators via handheld-to-handheld transfer or transferred from the computer to the calculator via TI-Connect™ CE software.

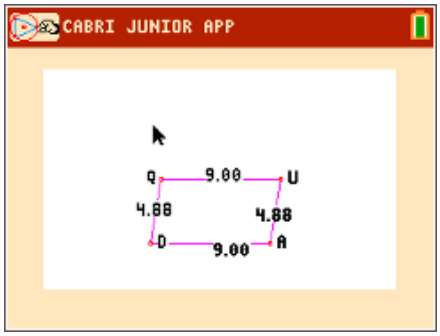
Problem 1 – Properties of Parallelograms

1. Define parallelogram.

Answer: A quadrilateral with both pairs of opposite sides parallel.

Students will begin this activity by looking at properties of parallelograms. They will discover that opposite sides are congruent.

Students can grab a point by moving the cursor over a point, and then pressing alpha. They can use the arrow keys to move the point. Pressing alpha again will release the point.



2. Open the file *PAR1* by pressing y=, selecting **Open…**and selecting the file. *PAR1* shows parallelogram *QUAD*. Grab and drag point *Q* to two different positions and record the lengths of the segments in the table (rows 1 and 2). Then, grab and drag point *U* to two different positions and record the data in the table (rows 3 and 4).

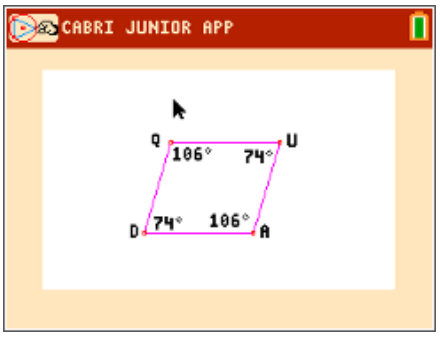
Sample Answers:

Position	$\overline{QU}$	$\overline{UA}$	$\overline{AD}$	$\overline{DQ}$
1	4.50	2.44	4.50	2.44
2	4.50	2.94	4.50	2.94
3	4.90	2.94	4.90	2.94
4	5.50	2.94	5.50	2.94

3. What do you notice about the lengths of opposite sides of a parallelogram?

Answer: The opposite sides are congruent.

Students will continue looking at properties of parallelograms. They will discover that opposite angles are congruent, and consecutive angles are supplementary.



4. Open the file *PAR2*, which shows parallelogram *QUAD*. Grab and drag point *Q* to four different positions and record the measurement of the angles in the table.

Sample Answers:

Position	∠*Q*	∠*U*	∠*A*	∠*D*
1	100	80	100	80
2	106	74	106	74
3	124	56	124	56
4	77	103	77	103

5. What do you notice about consecutive angles of a parallelogram?

Answer: The consecutive angles are supplementary.

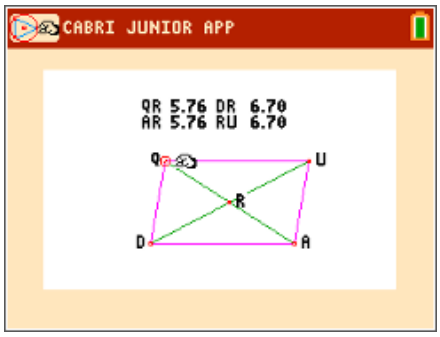
6. What do you notice about opposite angles of a parallelogram?

Answer: The opposite angles are congruent.

As an extension, students can prove each of these using parallel lines and transversals. Students will need to know the properties of alternate interior angles, same-side interior angles, and corresponding angles.

Problem 2 – Diagonals of Parallelograms

In Problem 2, students are asked to investigate the diagonals of a parallelogram. Students should discover that the diagonals of a parallelogram bisect each other. This particular wording may be hard for students discover independently.



7. Open the file *PAR3*, which shows parallelogram *QUAD*. Record the lengths of the segments in the table (row 1). Then, grab and drag point *U* to three different positions and record the data in the table (rows 2, 3, and 4).

Sample Answers:

Position	$\overline{QR}$	$\overline{RA}$	$\overline{DR}$	$\overline{RU}$
1	2.84	2.84	3.29	3.29
2	2.94	2.94	3.45	3.45
3	3.06	3.06	3.58	3.58
4	3.19	3.19	3.71	3.71

8. What do you notice about the diagonals of the parallelogram?

Answer: The diagonals bisect each other.

Problem 3 – Extension: Proving Parallelograms

In this problem, students can explore various properties and see if they guarantee that a quadrilateral is a parallelogram.

Students should know that the following prove that a quadrilateral is a parallelogram:

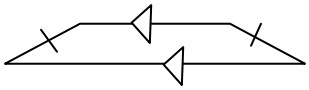
1) Both pairs of opposite sides are congruent
2) Both pairs of opposite angles are congruent
3) Both pairs of opposite sides are parallel
4) One pair of opposite sides is both parallel and congruent
5) The diagonals bisect each other

9. Does having both pairs of opposite sides congruent guarantee that the quadrilateral is a parallelogram? Draw an example or counterexample.

Answer: Yes

10. Does having one pair of opposite sides congruent and one pair of opposite sides parallel guarantee that the quadrilateral is a parallelogram? Draw an example or counterexample.

Answer: No. In a trapezoid, one pair of opposite sides is parallel and the other one pair of opposite sides could be congruent, but a trapezoid is not a parallelogram.

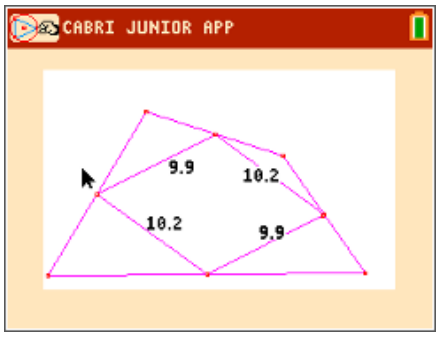


11. Does having one pair of opposite sides parallel and one pair of opposite angles congruent guarantee that the quadrilateral is a parallelogram? Draw an example or counterexample.

Answer: Yes

Problem 4 – Extension: Extending the Properties

For this problem, students will create any quadrilateral and name it *GEAR*. Next, students will find the midpoint of each side and connect the midpoints to form a quadrilateral. Students will use the properties of a parallelogram to see that a parallelogram is always created.



12. What type of quadrilateral is formed after you connected the midpoints of *GEAR*?

Answer: parallelogram

13. How can you prove what type of figure is created by connecting the midpoints?

Sample Answer: Both pairs of opposite sides are parallel.