

Triangle Areas

Teacher Notes & Answers

7 8 9 10 11 12



Introduction

In this activity you will explore ways to determine the area of a triangle.

Teacher Notes:

The area of a triangle is often presented as half the area of a rectangle; however, this is only obvious for right angled triangles. This activity leads students on a discovery that the area of a triangle is equal to half the product of the altitude and base. Teachers may like to reproduce a visual (instructions provided) to help students see the connection between the area of a triangle and a parallelogram.

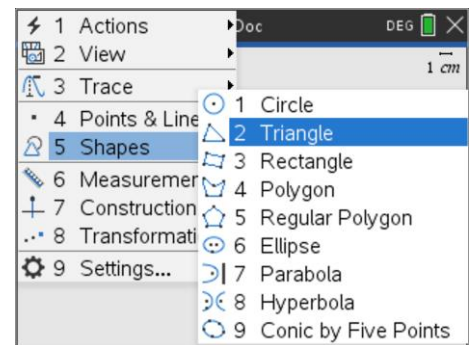
Exploring

Open a new TI-Nspire document and insert a Geometry Application.

From the menu:

menu > **Shapes** > **Triangle**

Draw a triangle by clicking to create a vertex and then moving the mouse to a new location, press ESC once you have created your triangle.

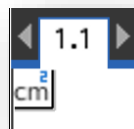


The next task is to measure the area of the triangle.

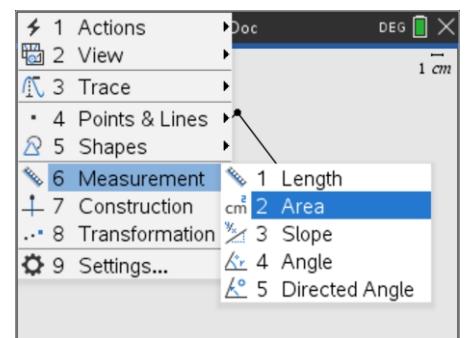
From the menu:

menu > **Measurement** > **Area**

With the measurement tool activated, select the triangle and the area will be displayed.



Note the measurement tool icon on the top left of the screen, press ESC to release the tool.

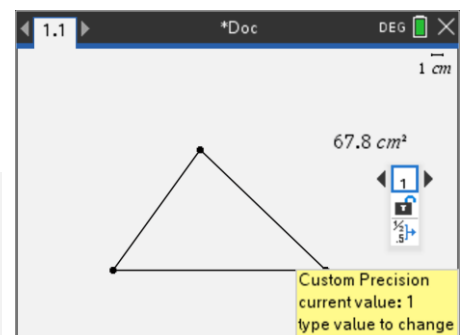


Drag the triangle vertices around on the screen and see how the area of the triangle is automatically updated.

Try and get the area as close to 100cm² as possible.



Navigate over the top of the measurement and press: **ctrl** + **menu** to change the precision. Use the left and right arrows on the navigation pad to increase or decrease the quantity of decimal places displayed.

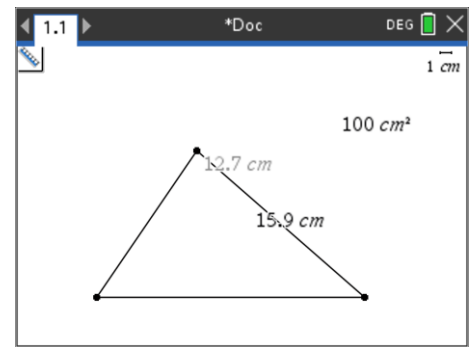


The next task is to measure the length of each side of the triangle.

From the menu:

menu > **Measurement** > **Length**

With the measurement tool activated, select one vertex and then the next. The length of the side will be displayed. Just like the area, these measurements are dynamic and update when the triangle is changed.



Question: 1

When the triangle has an area of close to 100cm^2 , record the three side lengths.

Answer: Answers will vary. Students will not be able to identify a rule from these measurements.

Teacher Notes: The purpose of this question is to check if students have created appropriate construction.

Teachers can check the student measurements by defining a function for Heron's formula.

Question: 2

Change the side lengths and try to create another triangle with an area of 100cm^2 .

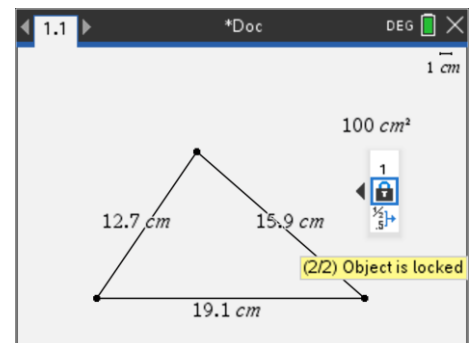
Is it possible to have different shaped triangles with the same area?

Answer: Yes – there are infinitely many triangles that will have an area of 100cm^2 .

Move the mouse over the measurement for the triangle area then press:

ctrl + **menu** > **Attributes**

Navigate down to the padlock, then use the right arrow key to lock the measurement. The area of the triangle is now locked.



Place the mouse over one of the triangle's vertices. The mouse will turn into a 'hand' and the tool tip "point" will be displayed. Press:

ctrl and click on the centre of the track pad.

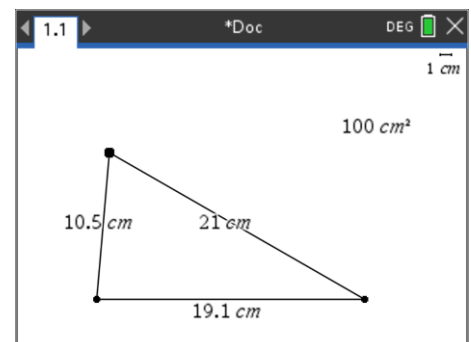
Alternatively, when the word "point" appears, click and hold the centre of the track pad. Swiping over the track pad now will attempt to move the triangle vertex (point).



To keep track of the movement of the vertex, press:

menu > **Trace** > **Geometry Trace**

Place the mouse over the top vertex and then grab and move it.



Question: 3

With the triangle area locked on 100cm^2 (approximately), describe how each vertex on the triangle moves.

Answer: Students should notice each vertex will move in a straight line, furthermore, the straight line should appear to be parallel to the side opposite the vertex.

Question: 4

As a vertex is moved, what measurements remain constant? Which measurements change?

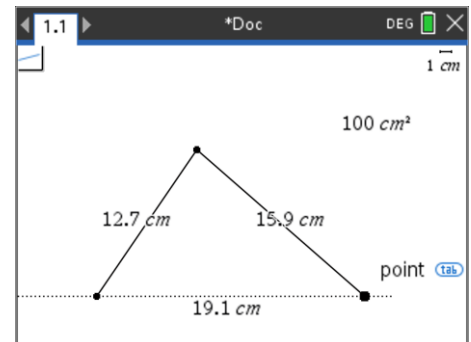
Answer: The length of the side opposite the vertex remains constant, the other two sides change in length.

Students may not realise that the height of the triangle remains unchanged also.

To help understand what is happening a couple of 'construction' lines need to be added to the diagram, press:

[menu] > Points & Lines > Line

Create a line that passes through the two vertices on the base of the triangle.

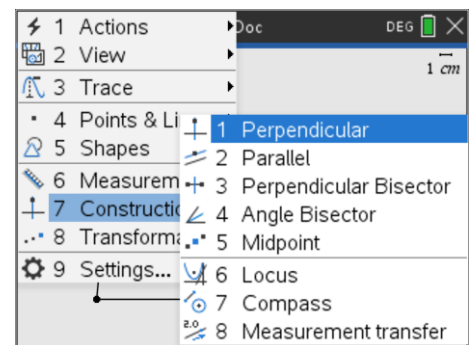


Now create a line perpendicular to the base of the triangle passing through the opposite vertex.

[menu] > Construction > Perpendicular

A perpendicular is a relationship between two lines such that the angle between them is 90° .

With the perpendicular tool active, place the mouse over the vertex closest to the top of the screen and click to select the vertex. Then move towards the opposite side.

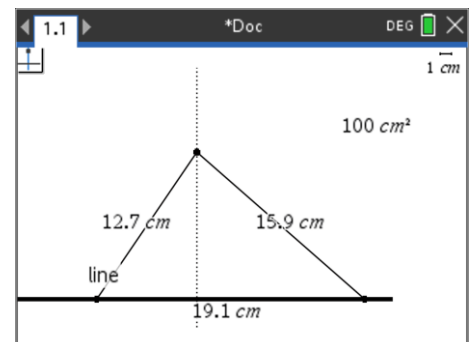


A line perpendicular to the 'base' of the triangle will appear as a dotted line, not that the tool tip must say "line". Click on the line to complete the construction of the perpendicular line.

Remember to press ESC to release the perpendicular construction tool.

Press P (point shortcut) and place a point where the altitude meets the base of the triangle. Watch for the tool tip: "Intersection Point".

A line perpendicular to the base, passing through the opposite vertex is referred to as an "**Altitude**". Think of the triangle as a pointy mountain, the base is the ground and the opposite vertex is the mountain peak or apex.

**Question: 5**

Measure the length of the altitude. Drag the top vertex around (triangle area still locked). Does the altitude of the triangle change?

Answer: The altitude of the triangle does not change when the area is locked.

Teacher Notes: Students now have the two values required to calculate the area of the triangle. Students can move the vertex so that the triangle is right-angled and note that it is 'half a rectangle'. However, showing students that the triangle is half of the parallelogram is a more powerful result.

Question: 6

Write down the length of your triangle base and the altitude. Multiply these values together and comment on your result.

Answer: Answers will vary for the side lengths, but the product of the sides should be double the triangle area.

Teacher Notes: As students all get individual results, this question allows teachers to check that the work being done by each student is their own.

Question: 7

Unlock the triangle area, then move the top vertex around to change the triangle area. Record the length of the base, triangle altitude and triangle area. Move the vertex around several times, record the three measurements each time and summarise your findings.

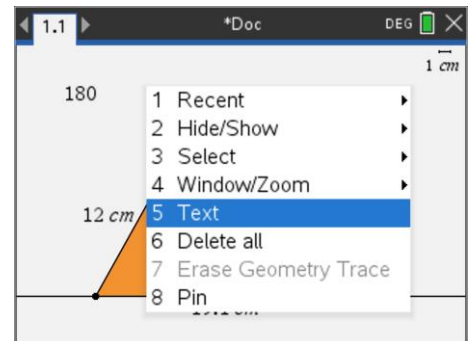
Answer: The result works for all triangles, not just those with an area of 100cm^2 .

Teacher Notes: The purpose of locking area to 100cm^2 previously was to make it easy for students to identify that the product of the altitude and base is double the triangle's area.

The instructions below illustrate how to show that the area of the triangle is half a parallelogram.

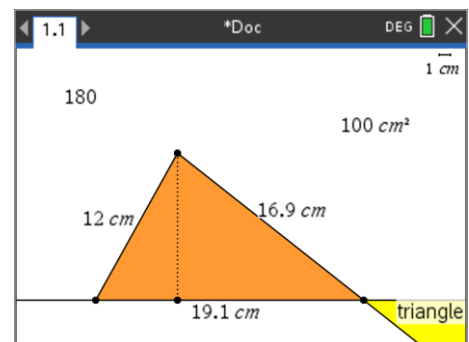
Use the attributes menu (Ctrl + Menu) and select **Text**.

Type 180 (degrees) as text in a blank region of the page.



Use the rotation tool in the Transformation menu. The rotation tool needs an object + point + angle.

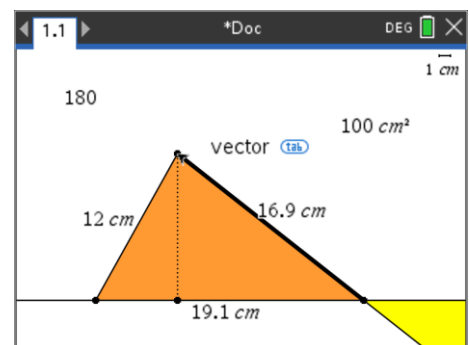
Select the triangle, the right hand vertex on the base of the triangle and then 180° .



Use the points and line tool to draw a vector from the bottom right vertex to the top of the triangle (shown opposite).

Finally, select the translation tool from the Transformation menu. The transformation tool needs an object (triangle) and vector (just created).

Select the triangle followed by the vector.



Hide the rotated triangle. The triangles shown opposite are 'congruent'. A rotation and translation in a plane are rigid body transformations, which means the distance between the vertices are unchanged. By SSS the two triangles are congruent.

The parallelogram is made up of two congruent triangles. If students know the formula for the area of a parallelogram as 'base x height' then it follows that the area of the triangle is $\frac{1}{2}$ x base x height.

