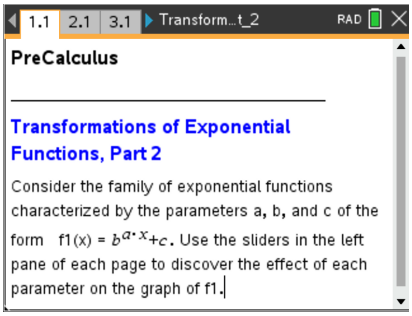
Transformations of Exponential Functions- Part 2 TEACHER NOTES

TI-NSPIRE™ CX/CXII TECHNOLOGY

Math Objectives

- Students will explore the family of exponential functions of the form $f(x) = b^{a \cdot x} + c$ and be able to describe the effect of each parameter on the graph of $y = f(x)$.
- Students will be able to determine the equation that corresponds to the graph of an exponential function.
- Students will understand that a horizontal dilation of the graph of an exponential function and a change of base of an exponential function are essentially the same.
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Vocabulary

- exponential function
- translation
- horizontal dilation
- parameter
- reflection
- change of base

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function
- entry line by pressing ctrl G

About the Lesson

- This lesson involves the family of exponential functions of the form $f(x) = b^{a \cdot x} + c$.
- As a result, students will:
  - Manipulate sliders, and observe the effect on the graph of the corresponding exponential function.
  - Conjecture and draw conclusions about the effect of each parameter on the graph of the exponential function.
  - Compare horizontal dilations and change of base and manipulate equations to demonstrate they are the same.
  - Match specific exponential functions with their corresponding graphs.

Lesson Files:

Student Activity

Transformations_of_Exponential_Functions_Part_2_Student.pdf

Transformations_of_Exponential_Functions_Part_2_Student.doc

TI-Nspire document

Transformations_of_Exponential_Functions_Part_2.tns

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Quick Poll to assess students' understanding.

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: To change a slider setting, right-click in the slider box, and select option 1. Consider changing the (start) value, minimum and/or maximum value, and/or the step size in order to help discover or confirm
the effect of a specific parameter.

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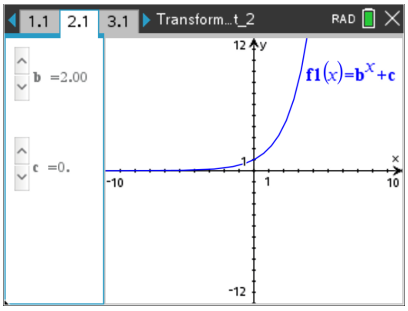
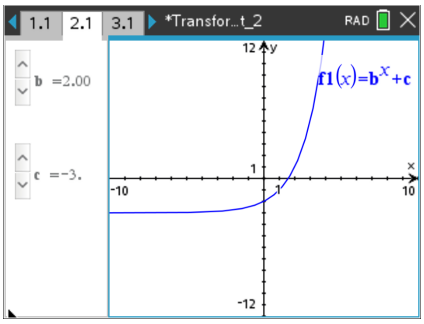


1. The graph of $y = f1(x) = b^x + c$ is shown in the right panel. For a specific value of b, click the arrows to change the value of c and observe the changes in the graph of $f1$. Repeat this process for other values of b.
   a. Explain why for every value of b, the graph of $f1$ passes through the point $(0, c+1)$.

Answer: The graph of $y = b^x$ passes through the point $(0,1)$ for all values of $b > 0$ because $b^0 = 1$. The graph of $y = f1(x) = b^x + c$ is the graph of $y = b^x$ with a vertical translation of c units and $f1(0) = b^0 + c = 1 + c$.



   b. Is it possible for the graph of $y = b^x + c$ to intersect the x-axis? Explain why or why not.

Answer:

The x-axis, the line $y = 0$, is a horizontal asymptote to the graph of $y = b^x$. If the graph of the function has a vertical translation of $-c$ units, the graph of the function would intersect the x-axis.

Possible example: $y = f1(x) = 2^x - 3$

Teacher Tip: The slider for the variable b is set to minimized, style:
vertical, and initially set such that it includes the value 1. Most definitions of an exponential function stipulate $b \neq 1$.

TI-Nspire Navigator Opportunity: ***Screen Capture and Quick Poll***
See Note 1 at the end of this lesson.

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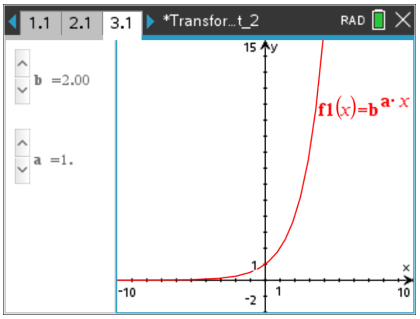

2. The graph of $y = f1(x) = b^{a \cdot x}$ is shown in the right panel. For a specific value of b, click the arrows to change the value of a and observe the changes in the graph of $f1$. Repeat this process for other values of b.
   a. Describe the effect of the parameter a on the graph of $y = b^{a \cdot x}$. Discuss the effects of both positive and negative values of a.

Answer:

The graph has a horizontal dilation. For $|a| > 1$, the graph of $y = b^{a \cdot x}$ is compressed horizontally by a factor of $\frac{1}{a}$. For $|a| < 1$, the graph of $y = b^{a \cdot x}$ is stretched horizontally by a factor of $\frac{1}{a}$.

If $a < 0$, the graph is reflected across the y-axis.

TI-Nspire Navigator Opportunity: ***Screen Capture and Quick Poll***
See Note 1 at the end of this lesson.

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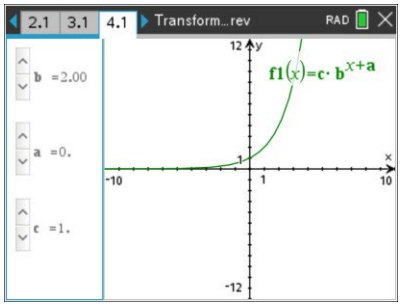

3. The graph of $y = f1(x) = b^{a \cdot x} + c$ is shown in the right panel. For specific values of a and b, click the arrows to change the value of c, and observe the changes in the graph of $f1$. Repeat this process for other values of a and b.
   a. Describe the effect of the parameter c on the graph of $y = f1(x) = b^{a \cdot x} + c$. Discuss the effects of both positive and negative values of c.

Answer:

The graph has a vertical translation. For $c > 0$, the graph of $y = b^{a \cdot x} + c$. is translated up. For $c < 0$, the graph of $y = b^{a \cdot x} + c$. is translated down.

TI-Nspire Navigator Opportunity: ***Screen Capture and Quick Poll***
See Note 1 at the end of this lesson.

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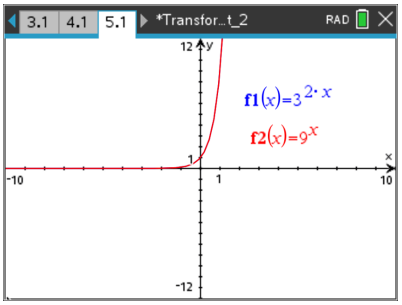
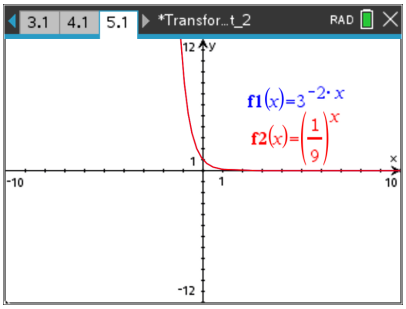
4. Display the graphs of $y = f1(x) = 3^{2x}$ and $y = f2(x) = 9^x$.



a. Describe the similarities between these two graphs. Use the properties of exponents to justify your answer.

Answer: The graphs of these two exponential functions are the same. $f1(x) = 3^{2x} = \left(3^2\right)^x = 9^x = f2(x)$.

b. Insert a new problem, and display the graph of $y = f1(x) = 3^{-2x}$. Use the properties of exponents to find a function of the form $f2(x) = b^x$ such that the graphs of $f1$ and $f2$ are the same. Verify your answer.



Answer: $f1(x) = 3^{-2x} = \left(3^{-2}\right)^x = \left(\frac{1}{9}\right)^x = f2(x)$.

The graphs of $f1(x)$ and $f2(x)$ are the same.

c. Use your answers to parts (a) and (b) to explain the relationship between a horizontal dilation of the graph of an exponential function and a change of base of an exponential function.

Answer: A horizontal dilation of the graph of an exponential function and a change of base are essentially the same. Consider the following expression to show this analytically.

$$f1(x) = b^{a \cdot x} = \left(b^a\right)^x = c^x = f2(x),$$

where $b^a =$ a constant, and $a \neq 0$. This demonstrates that any horizontal dilation can also be considered a change of base of exponential functions.

5. Without using your calculator, match each equation with its corresponding graph. Check your answers by graphing each function on your calculator.

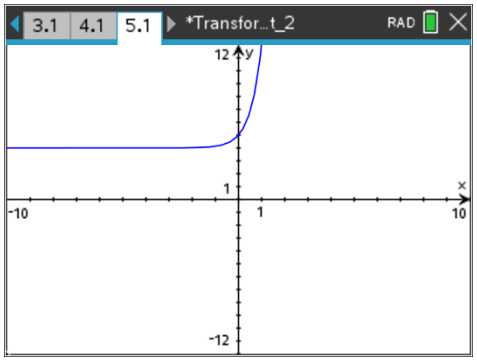
(a) $f(x) = 2^{3x}$ (b) $f(x) = -(2)^{3x}$

(c) $f(x) = 2^{-3x}$ (d) $f(x) = 2^{3x} + 4$

(e) $f(x) = e^{-x}$ (f) $f(x) = e^{x} - 3$

Note: The function in part (e) is the "natural" exponential function and involves the number $e \approx 2.71828...$

(i)



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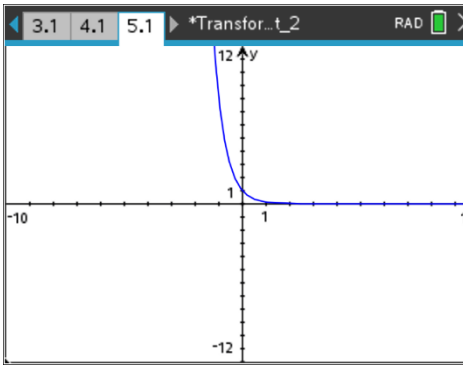

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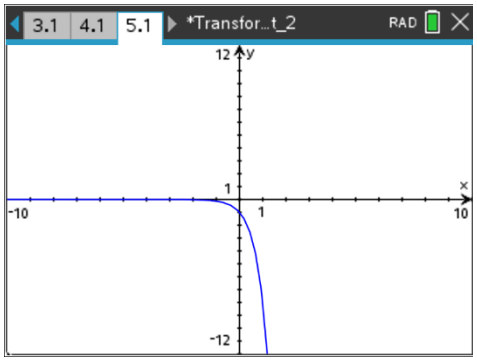

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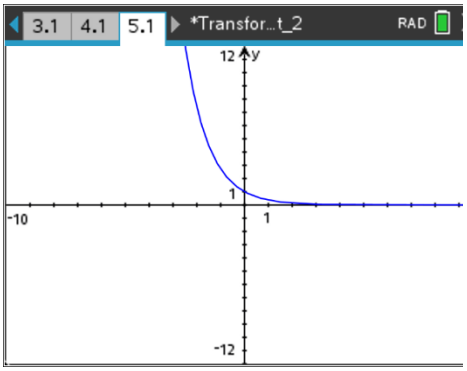

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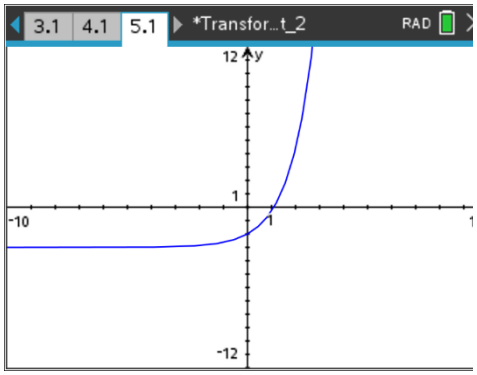

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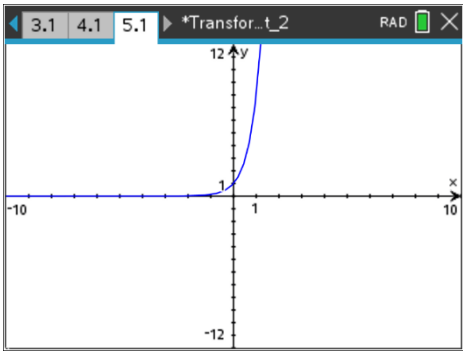

Answers: (a) (vi); (b) (iii); (c) (ii); (d) (i); (e) (iv); (f) (v).

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to:

- Graph and analyze an exponential function of the form $f(x) = b^{a \cdot x} + c.$
- Explain the concepts of dilation and translation.

TI-Nspire Navigator

Note 1

Name of Feature: Screen Capture and Quick Poll

Use Screen Capture to compare student graphs for various values of each parameter.

A Quick Poll can be given at several points during this lesson. It can be useful to save the results and show a Class Analysis.

Sample Multiple Choice questions.

For $b > 1,$ how many times does the graph of $y = 4^x + 2$ cross the x-axis?

(a) 0
(b) 1
(c) 2
(d) Infinitely many

Answer: (a)

How does the graph of $y = 4^x - 3$ compare to the graph of $y = 4^x$?

(a) Translated 3 units to the right
(b) Translated 3 units to the left
(c) Translated 3 units up
(d) Translated 3 units down.

Answer: (d)

Which of the following is equivalent to $y = 2^{-3x}$?

(a) $y = 8^x$
(b) $y = \left(\frac{1}{8}\right)^x$
(c) $y = 2^{-3+x}$
(d) $y = 2^{(1/3) \cdot x}$

Answer: (b)