

Radians

Student Activity

7 8 9 10 11 12



Introduction

Have you ever wondered why there are 360 degrees in a circle? What if we tried to make a 'metric' circle. For a metric circle we could argue for either 100 or 400 degrees, depending on whether we focus on a full revolution (100) or we consider 'right angles' (100) as the dominant reference. As it turns out this measure is already in existence, the measure is referred to as "Gradians". Another interesting consideration represents a movement away from rational numbers. As π is already entrenched in everything circle related, why not measure angles in terms of π ? As it turns out, this measurement is much more useful than degree and gradian measures.

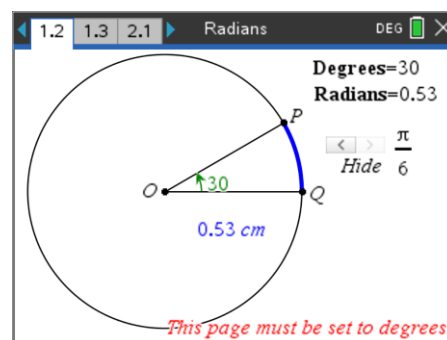
Introduction

Open the TI-Nspire document: **Radians**

Navigate to page 1.2.

A circle of radius one unit is displayed. Point P can be dragged around the circle. Angle $\angle QOP$ is measured in degrees, the arc length is displayed in centimetres (cm). The "Radian" measure will be exactly the same as the arc length for the unit circle.

An exact measurement for the arc length can be displayed or hidden.



Question: 1

What is the circumference of the circle?

Question: 2

Position point P to determine the radian measure for 180° ?

Question: 3

The exact radian measure for 30° is displayed as $\frac{\pi}{6}$, show how this result might be calculated by hand.

[Hint: Refer to Question 2]


Question: 4

Complete the table below for the conversion between degrees and radians.



Degrees	0	30	45	60	90	120	135	150	180
Radians									

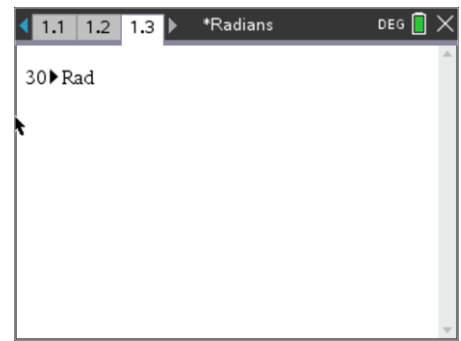
The document is currently set to degrees. [Top right corner of screen]

Navigate to page 1.3, a Calculator application. Type the value: 30. As the calculator is currently set to “DEG”, the value will be assumed as 30° if it represents an angle.

Use the catalogue key:  and press R, this will navigate to menu items starting with the letter R.

Navigate to ► **RAD**

Press  to accept the command and  again to execute.





Question: 5

What answer does the calculator provide when converting 30° to radians? [Include any symbols]

Question: 6

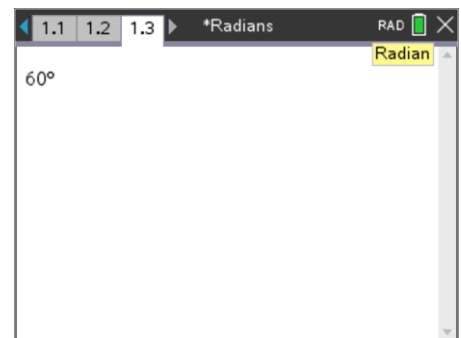
When the calculator is in degree mode it will automatically convert a radian angle into degrees. Enter $(\pi/4)$ and use the symbols template to add the radian symbol.

Note: The symbols template can be obtained by pressing:  + 

The document is currently set to degree mode. The mode can be changed via the document settings, or on the TI-Nspire CX II CAS, you can change the mode for the current application by simply clicking on the DEG symbol.

Change the document to RAD (Radians)

Enter an angle of 60° . Use the symbols template to access the degree symbol or the punctuation fly-out menu. When using the fly-out menu, you can continue to tap the key to scroll through the menu.



Question: 7

Convert 150° to radians using the calculator and explain how the answer relates to 30° and $\pi/6$.

Question: 8

Convert 210° to radians using the calculator and explain how the answer relates to 30° and $\pi/6$.

Question: 9

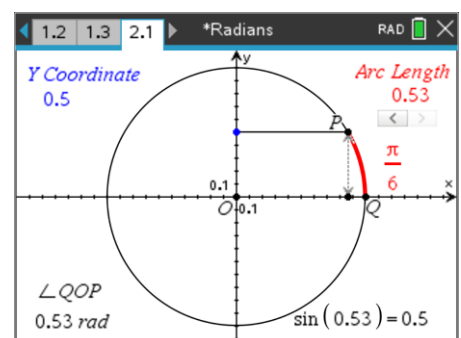
Convert 225° to radians using the calculator and explain how the answer relates to 45° and $\pi/4$.

Why?

Why do mathematicians bother with radians when degrees seem much easier? A clue lies in the trigonometric functions. Navigate to page 2.1 and once again, drag point P around the unit circle. This time the unit circle has been placed on the Cartesian plane. We are interested in two measurements:

- The length of the arc from Q to P.
- The distance from P to the x axis. (Y coordinate)

Note: The exact value for the arc length can be toggled off / on.



Question: 10

The diagram above shows $\sin(0.53) = \frac{1}{2}$ or $\sin(\pi/6) = \frac{1}{2}$. Find another arc length where this is also true.

Question: 11

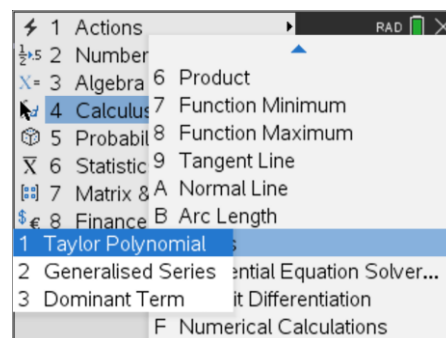
Find two arc length where the y coordinate is equal to $-\frac{1}{2}$. Comment on your findings.

The radian measure and its relationship with circular functions go much further.

Navigate back to page 1.3.

Define a function $s(x)$, this will be a special type of polynomial.

menu > **Calculus** > **Series** > **Taylor Polynomial**

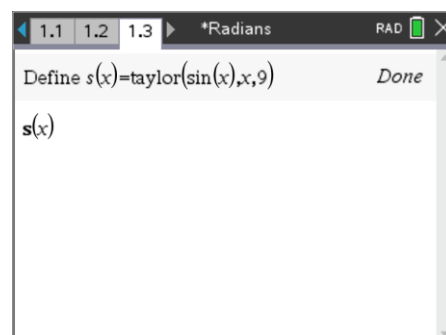


The syntax for the **Taylor** polynomial is:

Taylor (expression, variable, degree)

The expression being investigated is: $\sin(x)$, the variable is x and degree of the polynomial is 9. [It can be much higher, but this will do for now.]

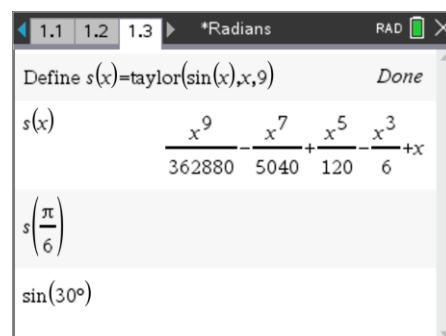
Once the expression has been defined type: $s(x)$



Determine the value of $s(\pi/6)$, press **ctrl** + **enter** to obtain an approximate answer.

Compare the result to $\sin(\pi/6)$ and $\sin(30^\circ)$, remember to use the degree symbol since the calculator should now be in radian mode.

Note: The answers have been removed from the screen opposite.

**Question: 12**

How do the answers compare?

Question: 13

Compare answers for $s(\pi/3)$, $\sin(\pi/3)$ and $\sin(60^\circ)$.

The Taylor polynomial is a function of infinite degree: $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} \dots$

Question: 14

What would be the next term in this sequence?

Question: 15

Using $x = \frac{\pi}{6}$ explain why the next term (Question 10), and subsequent terms are not critical for small angles.

Question: 16

Determine the Taylor polynomial for $\cos(x)$ and test some angles. [Make sure you are using radians.]