Midpoints Investigation

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Teacher Notes & Answers

7   8   9   **10   11**   12

TI-Nspire™   Investigation   Student   50 min

Introduction

In this investigation you will determine a formula for finding the midpoints of two given points and then apply this to find the midpoint of a fixed point and a point on a line.

Start a new document and insert a Graphs application. If the dot-grid is not displayed press:

> menu > Settings > dot grid

With the dot grid displayed, press P and select Point.

Place two points on the grid, one in quadrant one and the other in quadrant 2. Placing the points on the grid will ensure the points will be whole numbers.

Add the coordinates to both points:

> menu > Actions > Coordinates & Equations

Place point A at (-4, 3) and point B at (6, 3).

The next task is to automatically locate the midpoint of A and B.

> menu > Geometry > Construction > Midpoint

With the midpoint tool active, click on point A followed by point B, then get the coordinates of this point.

This resource can now be used to answer the following questions by dragging point A or B to the required location(s).
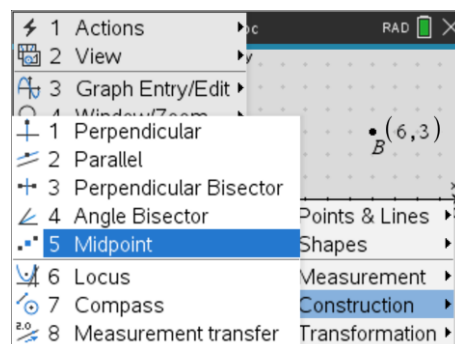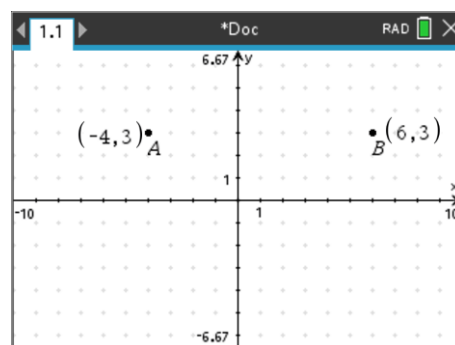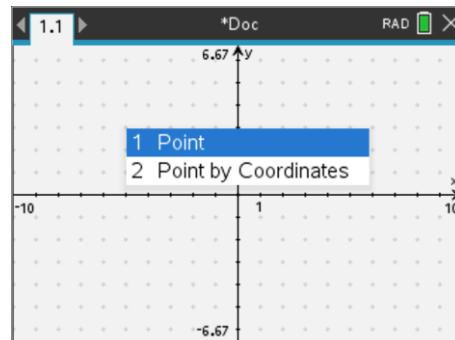
Question: 1.

Determine the coordinates of the midpoint for each of the following pairs of points:

a) (-4, 3) & (6, 3)   **Answer**: (1, 3)

b) (2, 3) & (6, 3)   **Answer**: (4, 3)

c) (2, -2) & (6, -2)   **Answer**: (4, -2)

d) (-6, 1) & (6, 1)   **Answer**: (0, 1)

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Question: 2.

Each pair of points in Question 1 has a different abscissa (x coordinate) and a common ordinate (y coordinate). Based on your results, suggest a method of calculating the abscissa for the midpoint.

Answer: "Calculate the average of the two x coordinates." $\dfrac{x_a + x_b}{2}$

Question: 3.

Determine the coordinates of the midpoint for each of the following pairs of points:

a) (2, 5) & (2, -3)   **Answer**: (2, 1)

b) (-3, 4) & (-3, -2)   **Answer**: (-3, 1)

c) (-5, 6) & (-5, -4)   **Answer**: (-5, 1)

d) (0, 5) & (0, -3)   **Answer**: (0, 1)

Question: 4.

Each pair of points in Question 3 has a different ordinate and a common abscissa. Based on your results, suggest a method of calculating the ordinate for the midpoint.

Answer: "Calculate the average of the two y coordinates." $\dfrac{y_a + y_b}{2}$

Question: 5.

Determine the coordinates of the midpoint for each of the following pairs of points:

a) (1, 4) & (7, 2)   **Answer**: (4, 3)

b) (-5, 4) & (7, 2)   **Answer**: (1, 3)

c) (-5, -4) & (3, 2)   **Answer**: (-1, -1)

d) (-5, -5) & (1, 5)   **Answer**: (-2, 0)

Question: 6.

Does your calculation method from Questions 2 and 4 work for any pair of points?

Answer: Yes … the general formula for the midpoint is: $\left(\dfrac{x_a + x_b}{2}, \dfrac{y_a + y_b}{2} \right)$

Question: 7.

Point A is located at (-4, -6), the midpoint is at the origin. Determine the location of point B.
Answer: (4, 6)

Question: 8.

Points A is located at (4, 4). The **midpoint** of point A and B is located on the y axis. Describe the location of point B.
Answer: Point B will be on the line $x = -4$

Teacher Notes: Students can determine this equation through an informal process of 'trial and error' or careful observation.

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Question: 9.

Point A is located at (3, -3). Point B is located on the line $y = x$. Describe the location of the midpoint.

Answer: The midpoint will be on the line $y = x - 3$

Teacher Notes: Students can determine this equation through an informal process of 'trial and error' or careful observation using the calculator. Students may also use an algebraic approach:

Let (x_m, y_m) represent the midpoint $\left(\dfrac{3+x_b}{2}, \dfrac{-3+y_b}{2} \right)$ and point B is such that: $y_b = x_b$.

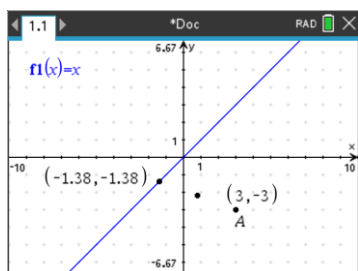We're required to find a relationship between x_m & y_m.

The midpoint can be re-written as: $\left(\dfrac{3+x_b}{2}, \dfrac{-3+x_b}{2} \right)$ which means: $x_m = \dfrac{3+x_a}{2}$ and $y_m = \dfrac{-3+x_a}{2}$

Use simultaneous equations to remove x_a and therefore establish a relationship between (x_m, y_m)

Substitute: $x_a = 2x_m - 3$ into $y_m = \dfrac{-3+x_a}{2}$ to get: $y_m = x_m - 3$ the point B is on the line: $y = x - 3$

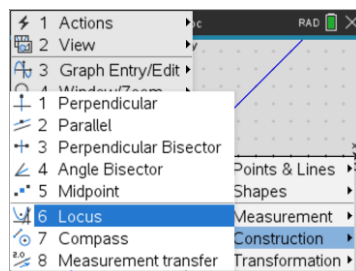Another option is for students to construct a locus, which also works well as a demonstration tool.

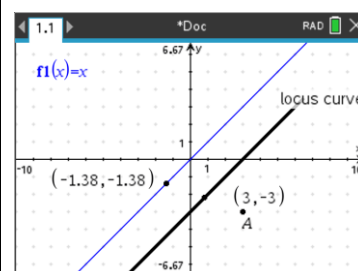| Graph the function and place point B on the function. Place point A at (3, -3) and use the construction tool to create the midpoint.	Use: Geometry > Construction > Locus Select Point B then midpoint.	Once the locus has been constructed, graph the function (answer) to show that the locus and linear function are one in the same.

Question: 10.

Points A is located at (4, 1). Point B is located on the line $y = x + 1$. Describe the location of the midpoint.

Answer: $y = x - 1$

Points A is located at (4, 1). Point B is located on the line $y = 2x + 1$. Describe the location of the midpoint.

Answer: $y = 2x - 3$

Extension:

Points A is located at (6, 2). Point B is located on the line $y = x^2$. Describe the location of the midpoint.

Answer: $y = 2(x-3)^2 + 1$

Teacher Notes: Students can determine this equation through an informal process of 'trial and error' or careful observation using the calculator. Students may also use an algebraic approach as per Question 9.

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