Math Objectives

- Students will use a table and a graph to compare the changes in linear and exponential expressions as *x* increases.
- Students will recognize that as *x* increases, a linear expression increases at a constant rate (additively) while an **exponential function** increases multiplicatively.
- Students will recognize that an exponential function with a positive base will never be less than or equal to 0, but will get smaller and smaller as *x* decreases.
- Students will determine whether a graph represents a linear or an exponential function.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

- exponential function

About the Lesson

- This lesson involves moving a point that changes the value of *x* and observing and comparing the values of a linear expression and an exponential expression.
- As a result, students will:
  - Compare linear and exponential expressions.
  - Compare linear and exponential functions.

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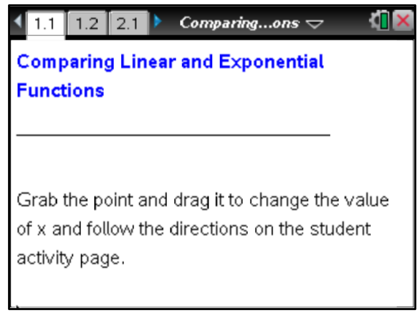
- Use Screen Capture to compare linear and exponential expressions.
- Use a Notes page and Screen Capture to compare and contrast linear and exponential functions.
- Use Quick Polls to assess students' understanding throughout the lesson.
- Use Teacher Edition computer software to review student documents.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Comparing_Linear_and_Exponential_Functions_Student.pdf
Comparing_Linear_and_Exponential_Functions_Student.doc

TI-Nspire document

Comparing_Linear_and_Exponential_Functions.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the point. Also, be sure that the word *point* appears. Then press ctrl to grab the point and close the hand ( ).

Teacher Note: This lesson can be used to probe more deeply into the behavior of exponential functions by changing the base in the .tns document, using numbers such as 2 or 0.5 for the base.

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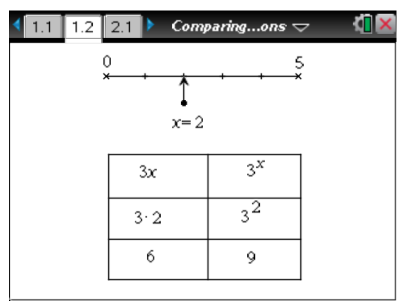
Use *Screen Capture* to determine whether or not students are experiencing difficulty using the .tns file. Use *Live Presenter* to demonstrate the correct procedure for using the file.

You may want to take a *Quick Poll* to see if most of the students are obtaining the correct answer to questions 2 through 4. This will enable you to either stop and clear up any misunderstandings, or continue with the lesson.

Move to page 1.2.



1. Grab and drag the point to change the value of *x*. Complete the table below. Which column is growing faster?

   Answer: The 3^x column is growing faster.

x	3*x*	3^x
0	0	1
1	3	3
2	6	9
3	9	27
4	12	81
5	15	243

2. a. As x increases from 2 to 3, how does the value of $3x$ change?

Answer: The value of $3x$ increases by 3.

b. As x increases by 1, describe the pattern in the numbers in the $3x$ column of the table.

Answer: The numbers increase by 3 each time.

> **Teacher Tip:** At this point, check for student understanding of repeated addition of 3.

c. As x increases from 2 to 3, how does the value of 3^x change?

Answer: It triples; it increases 3 times as much.

d. As x increases from 3 to 4, how does the value of 3^x change?

Answer: It triples; it increases 3 times as much.

e. As x increases by 1, describe the pattern in the numbers in the 3^x column of the table.

Answer: The numbers are being multiplied by 3. The values triple.

> **Teacher Tip:** Since the rate of change for $3x$ is constant, students might initially examine the values of 3^x in terms of rate of change. For instance, a student could respond "the value of 3^x increases by 18." In this case, you might ask the student if this pattern holds true for all changes in the value of 3^x. Since it does not, encourage the student to search for another pattern in the table.

3. On page 1.2 you can only look at values of x from 0 to 5. If x = 6, what would be the values of 3x and 3^x? How did you determine the values for $3x$ and 3^x?

Answer: Students might say that they added 3 to 15 (previous row) to get 18 and multiplied 243 by 3 to get 729, or any other acceptable method.

x	$3x$	3^x
6	18	729

4. Why are the values for 3^x increasing faster than the values for $3x$?

Answer: The values of 3^x are increasing faster than $3x$ because you multiply the previous number by 3 instead of adding 3 to the previous number. When a whole number greater than 1 is repeatedly multiplied by 3, the result gets greater faster than when you repeatedly add 3.

For example, if the whole number were 2, $2 \cdot 3 = 6$ while $2 + 3 = 5$. The product is greater at the beginning, and the sum will never catch up. $2 \cdot 3 \cdot 3 = 18$ while $2 + 3 + 3 = 8$.

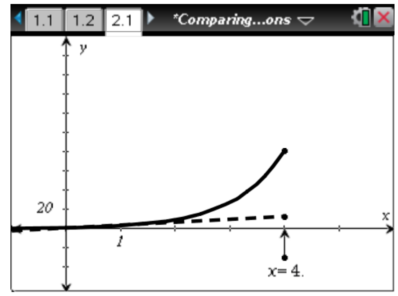
Teacher Tip: While multiplying whole numbers greater than 1 by a positive integer greater than 1 makes the product increase, students should recognize that when a fraction between 0 and 1 is multiplied by a constant multiplier greater than one, the results get smaller and smaller. For example, 1/3, 1/9, 1/27, and so on.

You might want to have students reflect on how multiplication works as repeated addition, that is $3 \cdot 2$ means two 3s or $3 + 3$. Thus, comparing 3^x to $3x$ going from $x = 5$ to $x = 6$ means for $3x$ you have five 3s or $3 + 3 + 3 + 3 + 3$ and the next term would have six 3s or $(3 + 3 + 3 + 3 + 3) + 3$ where you added a 3. With 3^5, the next term would be found by multiplying 3^5 by 3 or adding 3^5 three times: $3 \cdot 3^5 = (3^5 + 3^5 + 3^5)$. Two 3^5s were actually added to the previous term.

5. The function $f(x) = 3^x$ is called an ***exponential function,*** while the function $f(x) = 3x$ is a ***linear function.*** Describe the difference in the two functions.

Answer: A linear function has the variable as a factor in defining the function. In an exponential function, the variable is part of the exponent.

Move to page 2.1.



6. Drag the point to the right to produce two graphs—one solid, one dashed. Use the information from the table in question 1 to identify which graph represents an exponential function and which graph represents a linear function. Justify your answer.

Answer: The dashed graph remains closer to the x-axis and is $f(x) = 3x$ because it is increasing at a slower rate than the graph $f(x) = 3^x$. The graph of $f(x) = 3x$ increases at a constant rate, 3 units vertically for every 1 unit horizontally. The solid graph, $f(x) = 3^x$, increases at an increasing rate.

7. How do the graphs of $f(x) = 3x$ and $f(x) = 3^x$ support your response to question 4?

Answer: When comparing the y-values for $f(x) = 3x$, each time x increases by 1 unit, the y-value increases by 3 units. For $f(x) = 3^x$, each time x increases by 1 unit, the new y-value is 3 times the previous y-value.

8. Aaron says that the values of $f(x) = 5^x$ will increase faster than the values of the linear function $f(x) = 5x$. Do you agree or disagree? Justify your answer.

Answer: I agree with Aaron because for $f(x) = 5^x$, the y-values will be multiplied by 5 every time the x-value is increased by 1. For $f(x) = 5x$, 5 will be added to the previous y-value each time the x-value is increased by 1.

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Use *Quick Polls* to determine the number of students agreeing with the statement in question 8.

Teacher Tip: This might be a good time to ask students to give you examples of other linear or exponential functions.

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Have students press ctrl doc▾ and choose **Add Notes** to add a new notes page to the file. Have students compare and contrast linear and exponential functions on the page. Capture students' screens and discuss their responses.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- Expressions of the form $3x$ increase by repeated addition.
- Expressions of the form 3^x increase by repeated multiplication.
- Graphs of linear functions increase at a constant rate.
- Graphs of exponential functions of the form $y = b^x$, where b is greater than 1 increase faster than graphs of linear functions of the form $y = bx$.
- Exponential functions of the form $y = b^x$, where b is greater than 0 will never have values for $f(x)$ that are 0 or negative.

Extension: Trying Other Bases

Have students press ctrl G to show the function entry line on page 2.1. Then press the ▲ on the Touchpad twice to move to **f1**(x) and press the ◀ until the cursor is between the base and the exponent. Press del and change the base from 3 to 5. Press enter.

Have students press ctrl G again and press the ▲ on the Touchpad once to move to **f2**(x). Move the cursor until it is to the right of 3 and press del. Change the 3 to a 5. Press enter.

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Students then drag the point on the arrow to the right to see the two graphs. Use *Screen Capture* to view the screens. Was Aaron correct?

You might want to have different groups of students change the coefficient of the linear equation and the base on the exponential equation to other numbers greater than 1 and use *Screen Capture* to compare the results. Numbers between 0 and 1 can be used. Have students press **Menu > Window / Zoom > Zoom – Out >** enter before moving the point on the arrow to the left.