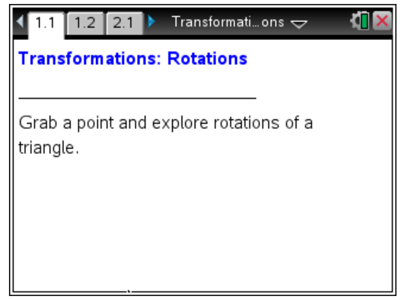
# Transformations: Rotations

**Math Nspired**




## Math Objectives

- Students will identify a rotation as an isometry, also called a congruence transformation.
- Students will identify which properties (side length, angle measure, perimeter, area, and orientation) of a figure are preserved in a rotation and which are not.
- Students will describe the relationship between the sign of the angle and the direction of rotation.
- Students will identify coordinates of an image that is rotated about the origin through angles of ±90°, ±180°, ±270°, and ±360°.
- Students will generalize the relationship between the coordinates of a pre-image and its image in a rotation about the origin in the coordinate plane.
- Students will look for and make use of structure and express regularity in repeated reasoning (CCSS Mathematical Practice).

## Vocabulary

- pre-image and image
- transformation
- rotation
- isometry
- clockwise / counterclockwise direction
- congruent figures
- congruence transformation
- positive and negative angles

## About the Lesson

- In this lesson students will investigate the meaning of a rotation, and they will discover which properties are preserved in a rotation and which are not. They will identify and generalize the coordinates of a triangle under rotations in the coordinate plane.
- As a result students will:
  - Rotate a triangle in clockwise and counterclockwise directions to develop their visualization and special sense of a rotation.
  - Describe the consequences of the rotation in terms of identifying those properties which are preserved and those which are not, and identify and generalize the coordinates of rotations in the coordinate plane.
  - Infer that a rotation does not alter any of the measurements of a rotated object and, as such, a rotation is an example of an isometry, or congruence transformation.

## TI-Nspire™ Navigator™

- Send the .tns file to students.
- Use Quick Poll questions to monitor student understanding.

## Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

**Tech Tips:**

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

**Lesson Files:**

*Student Activity*

- Transformations_Rotations_Student.pdf
- Transformations_Rotations_Student.doc

*TI-Nspire document*

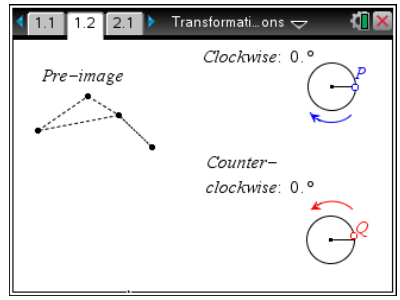
- Transformations_Rotations.tns

## Discussion Points and Possible Answers

**Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand ( ) getting ready to grab the point. Press ctrl to grab the point and close the hand ( ).

**Move to page 1.2.**




1. Determine if a rotation of a figure changes the size or the shape of the figure.
   a. Two figures are said to be *congruent* if they have the same size and same shape. Move point *P* on the *Clockwise* circle and observe the rotated image. Does the rotated image appear to be congruent to the pre-image? Why or why not?

   **Answer:** Yes, the rotated triangle appears to be the same size and shape as the pre-image triangle.

   b. Move point *Q* on the *Counterclockwise* circle and observe the rotated image. Does the rotated image appear to be congruent to the pre-image?

   **Answer:** Yes, the rotated triangle appears to be the same size and shape as the pre-image triangle.

2. An **isometry** is a transformation that produces an image that is congruent to the pre-image. Is a transformation by using a rotation an isometry? Explain your reasoning.

   **Answer:** Yes because the pre-image and the image are congruent.

**Teacher Tip:** This is an example of an isometry because it is a congruence transformation. The measurements of all sides and angles are preserved by the rotation. Since all of the pairs of corresponding parts are congruent, the triangles are congruent.

3. Move point *Q* (on the *Counterclockwise* circle) until the degree of rotation is $60^o$.
   a. If you move point *P* (on the *Clockwise* circle), when will the 2 rotated triangles be in the same position?

**Answer:** A clockwise rotation of $300^o$ produces the same image as a counterclockwise rotation of $60^o$.

   b. What do you notice about these 2 numbers?

**Answer:** Their sum is 360°.

4. Move point *P* (on the *Clockwise* circle) until the degree of rotation is $150^o$.

   a. If you move point *Q* (on the *Counterclockwise* circle), when will the 2 rotated triangles be in the same position?

**Answer:** A counterclockwise rotation of $210^o$ produces the same image as a clockwise rotation of 150°.

   b. What do you notice about the sum of these 2 numbers?

**Answer:** Their sum is 360°.

5. a. If the angle of the clockwise rotation of the pre-image is 135°, then what counterclockwise rotation will give you the same image? Why?

**Answer:** 225°, because 360 – 135 = 225.

   b. If the angle of the clockwise rotation of the pre-image is $n$°, then what counterclockwise rotation will give you the same image?

**Answer:** $(360 - n)°$

**TI-Nspire Navigator Opportunity: Quick Poll**
**See Note 1 at the end of this lesson.**

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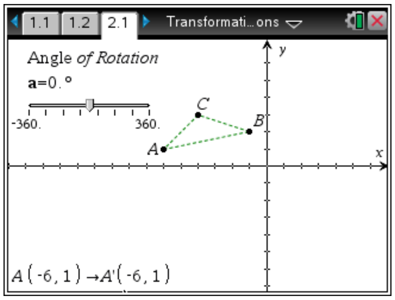


6. Use the slider to examine the difference between a positive and a negative angle of rotation.
   a. Start with the slider on an angle of rotation of $0^o$. Move the slider from $0^o$ toward $360^o$ and watch the image as it rotates. Does the triangle rotate in a clockwise or counterclockwise direction?

   **Answer:** A positive angle causes the triangle to rotate in counterclockwise direction.

   b. Start with the slider on an angle of rotation of $0^o$. Move the slider from $0^o$ toward $-360^o$ and watch the image as it rotates. Does the triangle rotate in a clockwise or counterclockwise direction?

   **Answer:** A negative angle causes the triangle to rotate in clockwise direction.

   c. What can you conclude about positive and negative angles of rotation and their relationship to clockwise and counterclockwise rotation?

   **Answer:** A positive angle causes the triangle to rotate in counterclockwise direction, and a negative angle causes the triangle to rotate in clockwise direction.

7. Use the slider to change the angle of rotation.
   a. Complete the first row of the table below with coordinates of vertex $A(-6, 1)$.
   b. Move vertex $A$ to a different location and record the new coordinates for each rotation in the second row of the table.
   c. Generalize your findings using the point $(x, y)$ in the third row of the table.

**Teacher Tip:** You might ask students to look for patterns in row c for angles that are even integral multiples of 90 and for angles that are odd integral multiples of 90.

| | | Coordinates of point *A′* | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | –360° | –270° | –180° | –90° | 0° | 90° | 180° | 270° | 360° |
| a. | (–6, 1) | **(–6, 1)** | **(–1, –6)** | **(6, –1)** | **(1, 6)** | (–6, 1) | **(–1, –6)** | **(6, –1)** | (1, 6) | **(–6, 1)** |
| b. | new point ( **–3, –2**) | **(–3, –2)** | **(2, –3)** | **(3, 2)** | **(–2, 3)** | **(–3, –2)** | **(2, –3)** | **(3, 2)** | **(–2, 3)** | **(–3, –2)** |
| c. | ($x$, $y$) | **($x$, $y$)** | (–$y$, $x$) | **(–$x$, –$y$)** | **($y$, –$x$)** | **($x$, $y$)** | **(–$y$, $x$)** | **(–$x$, –$y$)** | **($y$, –$x$)** | **($x$, $y$)** |

**TI-Nspire Navigator Opportunity: Quick Poll**
**See Note 2 at the end of this lesson.**

8. If the clockwise (or counterclockwise) order of the vertices of the image and the pre-image is the same, the figures are said to have the same **orientation.**

   a. Do △ABC and △A′B′C′ have the same orientation? Why or why not?

   **Answer:** Yes, they have the same orientation. The order of the vertices does not change.

   b. Does your answer to question 8a depend on the direction of rotation? Does it depend on the angle of rotation?

   **Answer:** No, I can rotate either direction and the orientation does not change. No, the angle of rotation does not change the orientation.

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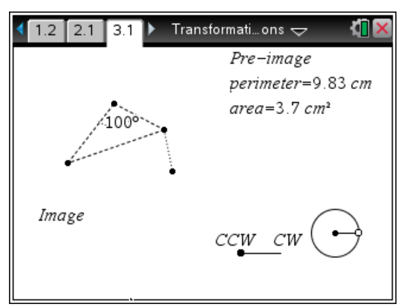


9. Use the point on the circle to change the angle of rotation. Use the slider to change from counterclockwise rotation to clockwise rotation.

   a. Record your observations as you change the angle and direction of rotation. What changes? What stays the same?

   **Answer:** The location of the image changes. Everything else (angle measure, side length, area, perimeter) stays the same.

**Teacher Tip:** When the direction of rotation changes from counterclockwise to clockwise, the actual visible rotation does *not* change. What changes is how the rotation angle measure is reported. If the angle of rotation on the circle shows a 60° angle and the direction is counterclockwise, the number displayed for the angle is indeed 60. If the direction is changed to clockwise, the number displayed is 300 (see figures). Help students to see that there are 2 angle/direction pairs that correspond to the same location of the image (as explored on page 1.2).

b. Move any of the vertices of the pre-image triangle. Does the new location of these points affect your observations in part a?

**Answer:** No, the location of the vertices of the pre-image does not affect the size, shape, or orientation of the rotated image.

**TI-Nspire Navigator Opportunity: Class Capture**
**See Note 3 at the end of this lesson.**

10. Consider the properties of side length, angle measure, perimeter, area, and orientation. Which of these properties are preserved in a transformation using rotation? How do you know?

**Answer:** All of these properties are preserved because the pre-image and the image have the same side length, angle measure, perimeter, area, and orientation.

## Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to:

- Identify a rotation as an isometry.
- Identify which properties of a figure are preserved in a rotation and which are not.
- Describe the relationship between the sign of the angle and the direction of rotation.
- Identify the coordinates of an image that is rotated about the origin through angles of ±90°, ±180°, ±270°, and ±360°.
- Generalize the relationship between the coordinates of a pre-image and its image in a rotation about the origin in the coordinate plane.

## TI-Nspire Navigator

**Note 1**

**Question 5, *Quick Poll*:** Send an open response Quick Poll to collect student responses to 5a and 5b.
You could also ask students for what degree rotation the clockwise and counterclockwise images will be the same.
**Answer:** 180°

**Note 2**
**Question 7, *Quick Poll*:** Send the following Quick Poll.
True/False: The image of (4, –3) rotated 270° is (–4, –3).
**Answer:** False. The image will be (*y*, –*x*), which is (–3, –4).

Point *A′*(3, –2) has a pre-image of *A*(–2, –3). What is the angle of rotation?
**Answer:** –270° or 90°.

**Note 3**
**Question 9, *Class Capture*:** Once students have moved the vertices of the pre-image triangle, use Class Capture so that students can verify their response to 9b.