Motorcycle Jump

ID: 8893

Time required
65-70 minutes

Activity Overview

This activity begins by presenting a scenario in which a motorcycle rider jumps off a ramp and travels along a quadratic path through the air. In Problem 1, students use a graphical model to explore the effect of setting the ramp at different angles to discover that the relationship between the angle of the ramp and the horizontal distance of the jump can also be described by a quadratic function. Students use this function to find the angle that maximizes the horizontal distance of the jump. In Problem 2, students relate the angle of the ramp and the airtime of the jump, and then they use a similar process to discover that the airtime of the jump increases without bound as the angle of the ramp approaches 90°. Finally, they use their results to make recommendations for the rider.

Topic: Quadratic Functions & Equations

- *Approximate the real zeros, vertex and extrema of a quadratic function graphically.*
- *Calculate the maximum and minimum value of a quadratic function.*
- *Use a quadratic function to model data.*

Teacher Preparation

- *This activity is designed to be used in an Algebra 2 or Precalculus classroom. Suggestions are given for more advanced students to explore the model in more depth. This activity could also be modified and used with Calculus students.*
- *This activity is intended to be mainly **teacher-led**, with some periods of independent student work.*
- *Prior to beginning this activity, students should have been introduced to quadratic equations and their graphs.*
- ***To download the calculator program MOTOR and the student worksheet, go to education.ti.com/exchange and enter "8893" in the keyword search box.***

Associated Materials

- *MotorcycleJump_Student.doc*
- *MOTOR.8xp (calculator program)*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *What is a Linear Regression? (TI-84 Plus family) — 4572*
- *Quadratic Regression with Transformation Graphing (TI-84 Plus family) — 8206*

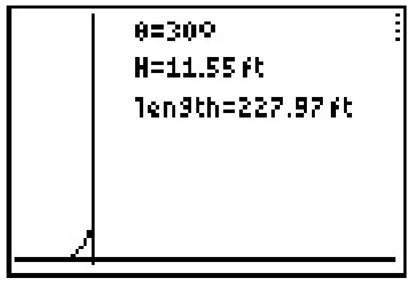
Problem 1 – Maximizing Horizontal Distance

In this problem, students use a model of the motorcycle jump to find the angle of the ramp that maximizes the (horizontal) length of the jump. Begin by introducing the problem scenario, printed on the student worksheet.

Take time to be sure that the students understand the scenario, and discuss the range of values that are possible for the angle of the ramp. Encourage them to hypothesize by asking questions such as: *What will the jump look like if the ramp is set at a steep incline? What if the ramp is set at a gentler incline?*

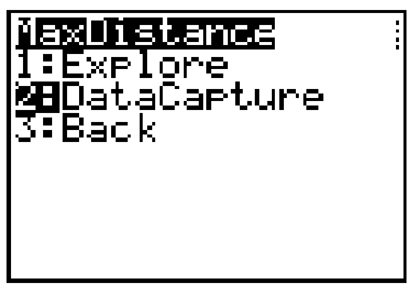
Then have students run the **MOTOR** program and choose **MaxDistance**, then **Explore**, and enter **30**.

The program shows the ramp and the path of the rider, then displays the angle measure, height of the top of the ramp, and length of the jump in feet. (**Note:** Due to window settings, the angle may not appear to be 30° on the screen.)



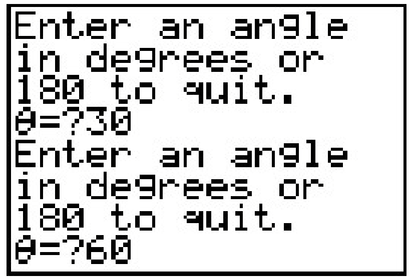
After making sure that students understand the model, allow students time to experiment with it independently (using only the **MaxDistance > Explore** portion of the program,) and answer questions 1–4 on their worksheet. They should observe that as θ increases from 0° to 90°, the length of the jump first increases and then decreases. Challenge students to use the model to approximate the value of θ that yields the jump with the longest length.

Having seen experimentally that there is a value of θ that maximizes the length of the jump, we now explore the math behind it by gathering data about the relationship between the angle of the ramp and the length of the jump. Demonstrate this process for the class first.

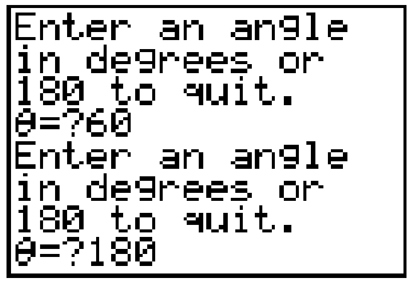


Go to **MaxDistance > DataCapture**, enter a value for θ, watch the jump, and press [ENTER].

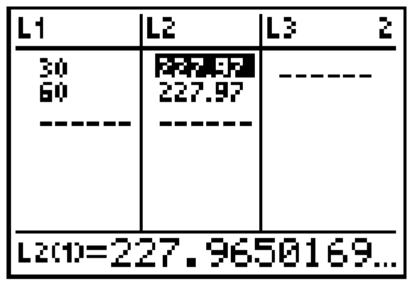
Enter another different value for θ and repeat.



Then enter **180** to exit the **DataCapture**. Choose **Back** and **Exit** to exit the **MOTOR** program.

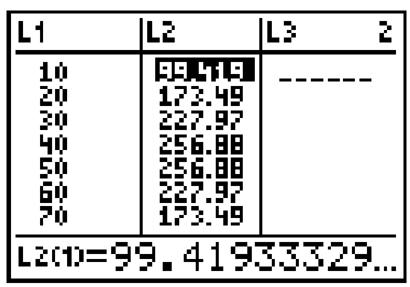


Press [STAT] [ENTER] to open the List Editor. The values of theta that you entered and the distances of the jumps are recorded in **L1** and **L2**.



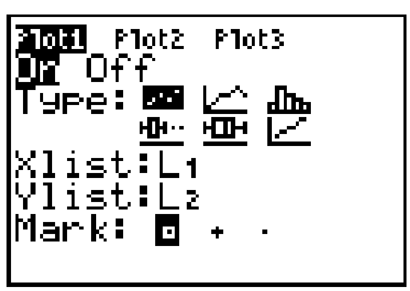
Have students perform their own data capture for at least 10 different values of theta. Caution them that if they exit the **DataCapture** and start over, the lists will be cleared and any data they had gathered will be lost.

Have students view **L1** and **L2** in the List Editor to confirm that they captured 10 data points. Ask: *Which is the independent variable,* θ *or length? Which is the dependent variable? Why?*



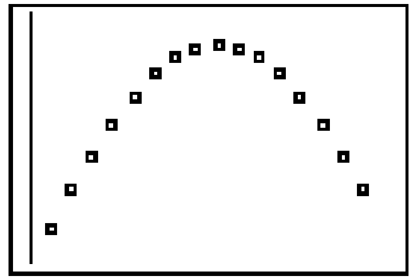
(The independent variable is θ, and the dependent variable is length, because the length of the jump is determined by θ.)

Guide students in creating a scatter plot of the data. Press [2nd] [STAT PLOT], then [ENTER] to open the settings for **Plot1**.



Set the plot to **On**, the **Type** to **Scatter**, the **Xlist** to **L1**, and the **YList** to **L2**.

Press [ZOOM] and choose **ZoomStat** to view the plot. Discuss the shape of the points as a class. The shape of this graph is similar to the model of the jump, so be sure to review what each axis represents and what is shown here.



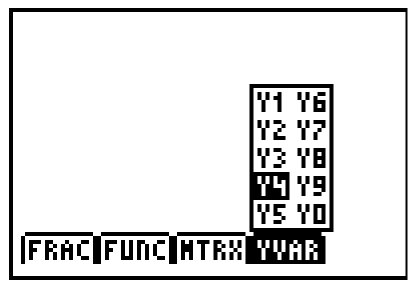
Ask: *Where is the length of the jump the greatest? The least? Is there a maximum point*?

Direct students to use their plots to answer questions 5–6 on their worksheets.

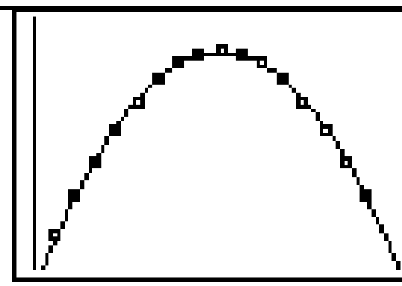
Help students perform a quadratic regression of their data. Go to [STAT] **> Calc > QuadReg** and then enter **L1**, **L2**, **Y4**. This calculates the regression and stores it in **Y4.**

If using Mathprint™ OS:

When entering the command for the linear regression, students can enter Y4 by either (1) press [VARS] [▸] [ENTER] and select Y4 or (2) press [ALPHA] [F4] and select Y4.



Press [GRAPH] to view the regression and data together. Discuss with students how they can use the regression equation to find the angle that maximizes the length of the jump.



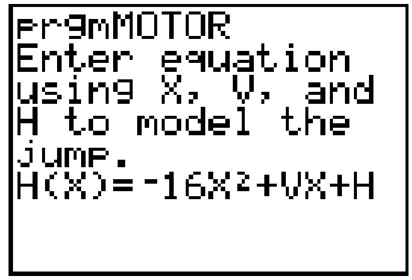
Ask: *Where on the parabola does the maximum occur? How can you find the coordinates of this point?*

Encourage students to determine that the *x*-coordinate of the vertex may be found using the formula $x = -\frac{b}{2a}$ and that the *y*-coordinate may then be found by substituting the *x*-coordinate into the regression equation. Have students answer questions 7–10.

Problem 2 – Maximizing Airtime

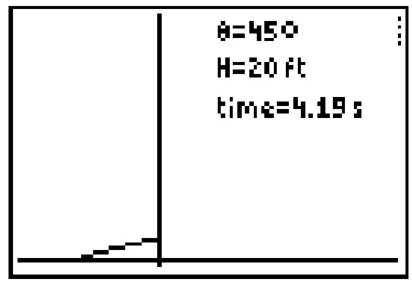
In this problem, students use the same process they used in Problem 1 with a quite different result. This time, though, the students are challenged to build the model themselves. This step is simplified greatly by the fact that the variables *V* and *H*, which depend on θ, are already stored in the handheld's memory. You might wish to have advanced students write the more complicated equation in terms of θ instead of *vy* and *h*, using the definitions of horizontal and vertical velocity.

Review the projectile motion equation if necessary. Then have students go to **MaxTime > SetModel** and enter and equation to model the height of the rider over time. The ramp provides a control of error. Their function should intersect the *y*-axis at the same point as the ramp.

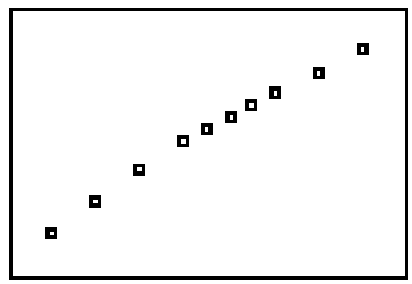


Check that all students have the correct model ($\mathbf{-16X^2 + VX + H}$) before proceeding. (**<u>Note</u>:** If students attempt to explore or capture data with an incorrect model, the calculator may display an error message. They should simply choose **Quit** and try another model.)

Allow students some time to experiment with different values of θ (**MaxTime > Explore**) and answer questions 11–15 on their worksheets. Discuss the results as a class. Students should notice that as θ approaches 90°, the longer the time of the jump, but that when θ = 90º, the airtime is undefined.



Students should then repeat the process of capturing data values (**MaxTime > DataCapture**) and plotting the values as a scatter plot.



Discuss the shape of the scatter plot with the class, and direct them to answer questions 16–18 on their worksheets. They should determine that this curve increases to infinity and does not have a maximum point.

Solutions

Problem 1

1. The "best" value of θ should be close to 45°.
2. The horizontal axis represents horizontal distance in feet.
3. The vertical axis represents vertical distance in feet. So the model is effectively a picture of the jump.
4. The model is not completely realistic because it would not be possible to ride up a perfectly vertical (or near vertical) ramp (when θ = 90°).
5. The scatter plot is shaped like a parabola.
6. There is an angle that maximizes the length of the jump, because the graph of angle vs. length has a maximum point.
7. Answers will vary because of different data, but should be similar to $y = -0.12x^2 + 11.54x - 0.21$.

8. Answers will vary, but should be close to 45°.

9. Answers will vary, but should be close to 260 feet.

10. The rider should set up the ramp at a 45° angle.

Problem 2

11. $y = -16x^2 + V \cdot x + H$ or $y = -16x^2 + (88 \cos\theta) \cdot x + (20 \tan\theta)$

12. The horizontal axis represents time in seconds. (So in this case, *x* is really *t*.)

13. The vertical axis represents vertical distance of height in feet.

14. $x = 0$ represents the point in time when the rider leaves the ramp.

15. As θ approaches 90°, the longer the time of the jump, but that when $\theta = 90°$, the time of the jump is undefined.

16. The plot shows a positive relationship between θ and the airtime of the jump.

17. There is no value of θ that maximizes airtime because the graph of angle versus airtime does not have a maximum.

18. The rider should set the ramp as close to 90° as possible such that she can still (and safely) ride up it at 60 mph.