



Concepts

These documents are designed to strengthen understanding of key concepts associated with the rectilinear, or straight line, motion of an object and to use these concepts to predict, or reason, backward, graphically and analytically. An important idea that is often difficult for students is the difference between total distance traveled and displacement. These documents help to visualize and reinforce this distinction.

The Velocity Position Distance documents allow the user to manipulate a piecewise linear function or to define an arbitrary function that represents the velocity of an object moving along a straight, horizontal line. The user may also set the initial position of the object, and the corresponding position function is dynamically displayed.

A trail of the path of the object is also displayed so that one can distinguish between the current position of the object, the displacement of the object, and even the speed of the object. Various associated numerical values are displayed in a separate pane. The position of the object is changed by using the clicker (a minimized slider), connected to the value of time t , in the left pane.

Course and Exam Description Unit

Section 4.2: Straight-Line Motion: Connecting Position, Velocity, and Acceleration

Calculator Files

- Velocity_Position_Distance_PT1.tns
- Velocity_Position_Distance_PT2.tns



Velocity, Position, Distance

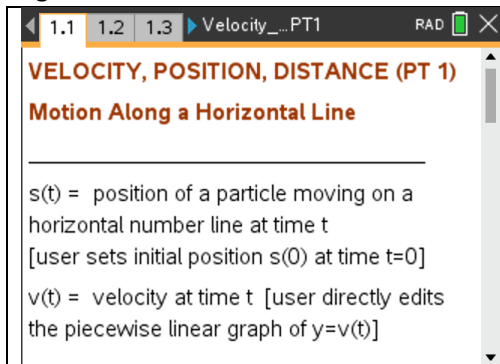


Using the Documents

The two calculator files differ in how the user sets, or defines, the velocity function.

Velocity_Position_Distance_PT1.tns:

Page 1.1



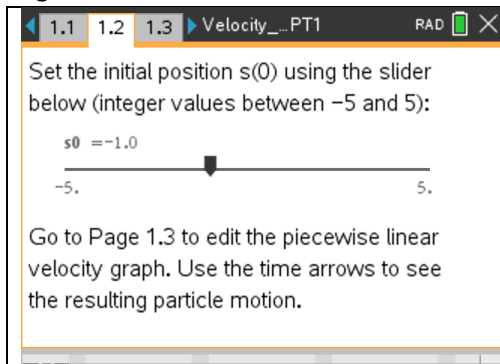
VELOCITY, POSITION, DISTANCE (PT 1)
Motion Along a Horizontal Line

$s(t)$ = position of a particle moving on a horizontal number line at time t
[user sets initial position $s(0)$ at time $t=0$]

$v(t)$ = velocity at time t [user directly edits the piecewise linear graph of $y=v(t)$]

The functions and variables used in this file are defined here. There is also a brief description of the graphs presented on pages 1.3 and 1.4. Note that the calculator variables used for displacement and total distance traveled are dsp and $disp$, respectively.

Page 1.2



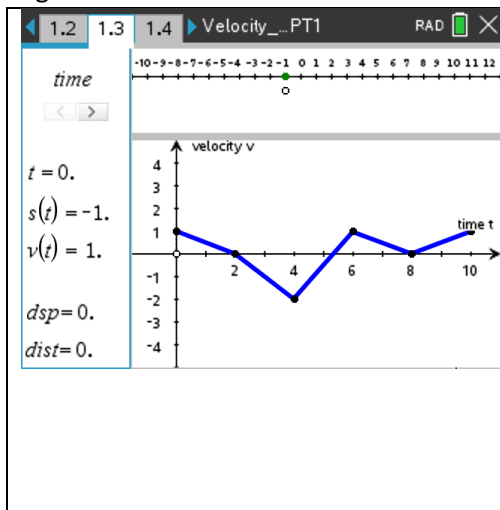
Set the initial position $s(0)$ using the slider below (integer values between -5 and 5):

$s_0 = -1.0$

Go to Page 1.3 to edit the piecewise linear velocity graph. Use the time arrows to see the resulting particle motion.

The user can set the initial position of the object to any integer value between -5 and 5 . The default initial position of the object is $s(0) = -1$.

Page 1.3

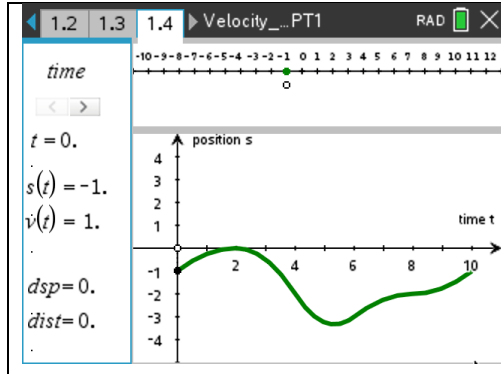


$t = 0.$
 $s(t) = -1.$
 $v(t) = 1.$
 $dsp = 0.$
 $dist = 0.$

The graph of the velocity of the object for $0 \leq t \leq 10$ is displayed in the bottom right pane. The vertices that connect the linear pieces of the graph can be moved up or down (in integer steps) by grabbing the point and dragging to another location. Values of t , $s(t)$, $v(t)$, the displacement and total distance traveled are presented in the left pane. The motion of the object is illustrated as the value of t is changed, either by using the *clicker* (minimized slider) or by grabbing and dragging the point on the horizontal axis corresponding to t . In the top pane, the green point represents the position of the object, the green line segment represents the displacement of the object, and the path of the object is represented just below the horizontal axis.



Page 1.4



The graph of the position of the object for $0 \leq t \leq 10$ is displayed in the bottom right pane. Values of t , $s(t)$, $v(t)$, the displacement and total distance traveled are presented in the left pane. The motion of the object is illustrated as the value of t is changed, either by using the clicker or by grabbing and dragging the point on the horizontal axis corresponding to t . In the top pane, the green point represents the position of the object, and the path of the object is represented just below the horizontal axis.

Suggested Applications and Extensions

Velocity_Position_Distance_PT1.tns

Use the default velocity function. On page 1.3, observe the numerical values in the left pane and the motion of the particle in the top pane as t increases from 0 to 10. The graph of the position of the object is given on page 1.4. Use these graphs to answer the following questions.

1. Use the velocity function to explain when the object is moving to the right. To the left.
2. Use the velocity function to explain when the object changes direction.
3. When is the object moving fastest? Slowest? Not at all?
4. Find the time at which the object is farthest from its starting point. What is the value of the velocity at that time?
5. Find the average velocity of the object over the time interval $0 \leq t \leq 10$.
6. When is the object speeding up? Slowing down?
7. Describe the difference between total distance traveled and displacement of the object at any time t .
8. Explain why the values of the total distance traveled and displacement are not the same for all values of t . When are the same for all values of t ?
9. Explain how the graph of the motion in the top pane (the trace of the motion in black) can be used to determine when the object is moving fast and when is it moving slowly.
10. For a specific value of t , explain how to use the graph of the velocity function to determine whether the speed of the object is increasing or decreasing.
11. Use values given in the left pane to estimate the acceleration of the particle at time $t = 5$.
12. Explain why the acceleration of the particle does not exist at time $t = 4$.
13. Find the position of the object at time $t = 6$. Explain how to use the velocity graph to find this value. Write an expression involving an integral that gives the position of the object at time $t = 6$.



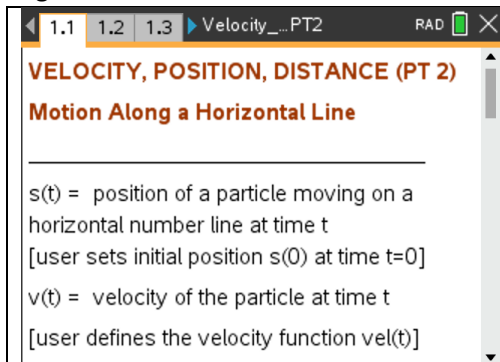
Velocity, Position, Distance

The following questions involve manipulation of the velocity graph on Page 1.3.

1. Manipulate the velocity graph so that after 10 seconds the object is as far away from the starting point as possible. What is this distance?
2. For $0 \leq t \leq 10$, construct a velocity graph such that the displacement is 0 and the total distance traveled is greater than 0.
3. Is it possible to construct a velocity graph such that the displacement and total distance traveled are both 0? If so, how?
4. Construct a velocity graph such that the particle changes direction twice and has total distance traveled for $0 \leq t \leq 10$ equal to 16.
5. Is it possible to construct a velocity graph such that the object changes directions 5 times for $0 \leq t \leq 10$? If so, how?
6. Construct a velocity graph such that the object has velocity 0 twice in the interval $0 \leq t \leq 10$, but is always moving in the same direction.

Velocity_Position_Distance_PT2.tns:

Page 1.1



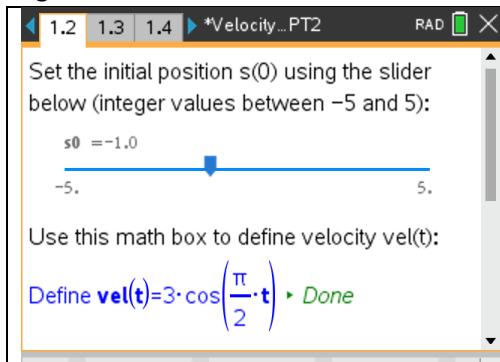
VELOCITY, POSITION, DISTANCE (PT 2)
Motion Along a Horizontal Line

$s(t)$ = position of a particle moving on a horizontal number line at time t
 [user sets initial position $s(0)$ at time $t=0$]

$v(t)$ = velocity of the particle at time t
 [user defines the velocity function $vel(t)$]

The functions and variables used in this file are defined here. There is also a brief description of the graphs presented on pages 1.3 and 1.4. Note that the calculator variables used for displacement and total distance traveled are dsp and disp, respectively.

Page 1.2



Set the initial position $s(0)$ using the slider below (integer values between -5 and 5):

$s_0 = -1.0$

Use this math box to define velocity $vel(t)$:

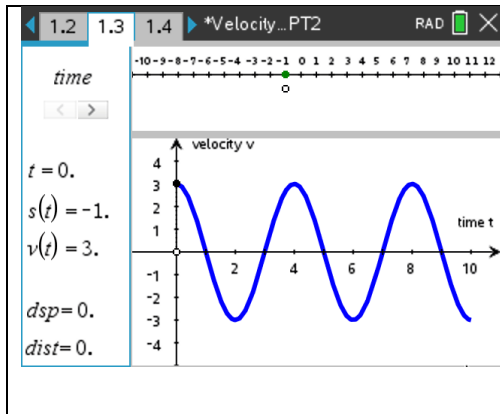
Define $vel(t) = 3 \cdot \cos\left(\frac{\pi}{2} \cdot t\right)$ Done

The user can set the initial position of the object to any integer value between -5 and 5 . The default initial position of the object is $s(0) = -1$. The expression for the velocity function can also be changed on this page, within a Mathbox. The default function is shown.



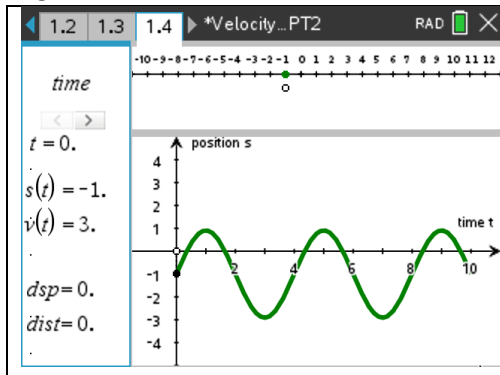
Velocity, Position, Distance

Page 1.3



The graph of the velocity of the object for $0 \leq t \leq 10$ is displayed in the bottom right pane. Values of t , $s(t)$, $v(t)$, the displacement and total distance traveled are presented in the left pane. The motion of the object is illustrated as the value of t is changed, either by using the *clicker* (minimized slider) or by grabbing and dragging the point on the horizontal axis corresponding to t . In the top pane, the green point represents the position of the object, the green line segment represents the displacement of the object, and the path of the object is represented just below the horizontal axis.

Page 1.4



The graph of the position of the object for $0 \leq t \leq 10$ is displayed in the bottom right pane. Values of t , $s(t)$, $v(t)$, the displacement and total distance traveled are presented in the left pane. The motion of the object is illustrated as the value of t is changed, either by using the *clicker* or by grabbing and dragging the point on the horizontal axis corresponding to t . In the top pane, the green point represents the position of the object, and the path of the object is represented just below the horizontal axis.



Suggested Applications and Extensions

Velocity_Position_Distance_PT2.tns

Use the default velocity function. On page 1.3, observe the numerical values in the left pane and the motion of the particle in the top pane as t increases from 0 to 10. The graph of the position of the object is given on page 1.4. Use these graphs to answer the following questions.

1. Use the velocity function to explain when the object is moving to the right. To the left.
2. Use the velocity function to explain when the object changes direction.
3. When is the object farthest to the right?
4. Find the average velocity of the object on the interval $[0,5]$. Estimate a value of t_a in the interval $[0,5]$ such that the $v(t_a)$ is equal to the average velocity. On page 1.4, use the built-in calculator functions to draw the line segment connecting the points $(0, s(0))$ and $(5, s(5))$. Draw the tangent line to the graph of s at the point $(t_a, s(t_a))$. How are these two lines similar?
5. Find the average speed of the object on the interval $[0,5]$.
6. Find the first value of t for which the distance traveled by the object is different from the displacement of the object. Use the graph of the velocity to explain why this has occurred.
7. Use the values in the left pane to estimate the acceleration of the particle at time $t = 3$.
8. Use the graph of the velocity to determine whether the object is speeding up or slowing down at time $t = 2$. Explain how the path of the particle in the top pane supports your conclusion.
9. On page 1.3, use the built-in calculator functions to evaluate $\int_0^4 v(t)dt$. Explain how this value is related to $s(4)$. Drag to the right the point on the graph corresponding to $(4,0)$. Observe the values of s and v , and write an expression involving a definite integral relates the functions s and v .
10. Find the total distance traveled at times $t = 5$ and $t = 10$. How are these values related? This suggests that the object travels half the total distance in half the time. Define a velocity function such that this is false.

Set the initial position to 0.

1. Find a velocity function such that the distance traveled is equal to displacement at $t = 10$. Explain why this is true for your function.
2. Find a velocity function such that the displacement is equal to (-1) times the distance traveled.
3. Find a velocity function such that at $t = 10$ the displacement is 0 but the distance traveled is nonzero.
4. Find a velocity function such that the object changes direction once for $0 \leq t \leq 10$.
5. Find a velocity function such that the object changes direction twice and has negative displacement for $0 \leq t \leq 10$.