Introduction

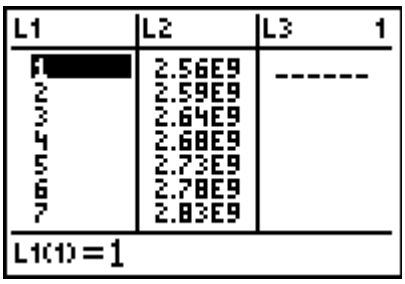
Population growth can be modeled using various exponential functions.

Objective: Use two different methods to find the best model for given data.

Data: midyear world population from the years 1950-2006 (year 1 represents year 1950)

Problem 1 – Find an exponential equation by hand using two points.

In this part, you will find an exponential equation using two points. Begin observing the data.

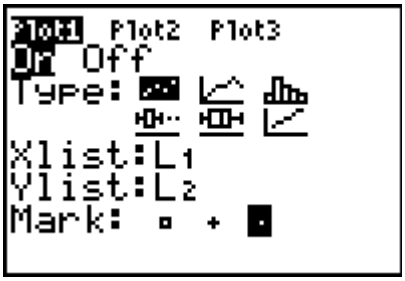


Press PRGM and choose **WORLDPOP**. When the screen displays **Done**, the world population data will be loaded into lists L1 and L2.

Press STAT ENTER to view the data.

- What kind of function would best model the data?

To examine the data, make a scatter plot.



Press 2nd [STAT PLOT] and select **Plot 1**. Adjust the settings to those shown at the right. Press ZOOM and select **ZoomStat** to view the plot in an appropriate window.

Now find an exponential model to fit the data by using two points (*x1*, *y1*) and (*x2*, *y2*). To do this, press TRACE to see the coordinate values. Pick two points that are spread throughout the data and write the values in the space provided:

- *x1* = _____ *y1* = _____
- *x2* = _____ *y2* = _____

The general formula for an exponential function is $\boldsymbol{y = a(b^x)}$ where *a* is the initial value and *b* is the multiplier or in this case the growth rate.

To find the equation through the two points, first, substitute the values into the general exponential formulas:

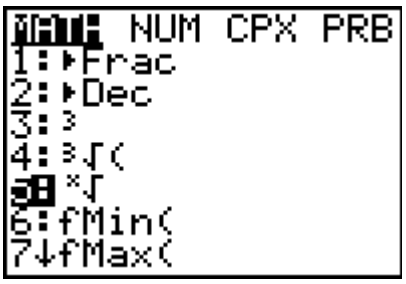
- $y1 = a(b^{x1})$ _____ = _____
- $y2 = a(b^{x2})$ _____ = _____

Then divide the equations and solve for *b*.

$$\frac{y2}{y1} = \frac{a\left(b^{x2}\right)}{a\left(b^{x1}\right)}$$ _____

- What happens to the value of *a* when the two equations are divided?

Subtract the exponents which are the *x*-values and divide the *y*-values using the formula $\sqrt[(x2-x1)]{\frac{y2}{y1}}$.



To find the *n*th root on the calculator, press MATH and select $\sqrt[x]{\ }$.

- What is your value of *b*?

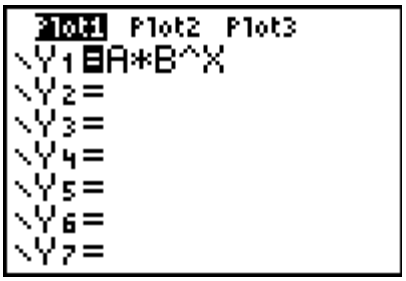
Store this value by pressing STO▸ ALPHA **B** ENTER.

- According to your *b* value, by what percent is the world population growing?

Now substitute the value of *b* back into either of the equations above to find the value of *a*. Follow the steps above to store this value as **A**.

- What is your value of *a*?
- What is your exponential equation?
- Look at your equation, what is your initial population?

To graph your equation with the data, press Y= and enter the expression shown at the right. Press GRAPH.



- How well does your equation model the data?
- What will the population be in the year 2015?
- What was the population was in the year 1890?

Problem 2 – Finding the exponential regression.

Now you will find an exponential regression equation to fit the data.

Press [STAT], move to the **CALC** menu and select **ExpReg**. Then enter the rest of the expression as shown at the right.

This lists the values for *a* and *b* in the general formula $y = a(b^x)$.
Press [GRAPH] to see both equations and the scatter plot.

- What is the exponential regression equation?
- What is the initial population?
- By what percent is the world population growing?
- How do these two values compare with the ones in Problem 1?
- How well does the exponential regression equation model the data?
- What will the population be in the year 2015?
- What was the population in the year 1890?
- Which model do you feel is best? Why?
- How would you find out what year the world's population will reach 8.5 billion people?
- Graphically use the exponential regression equation to find the year.