

Circles on the Complex Plane



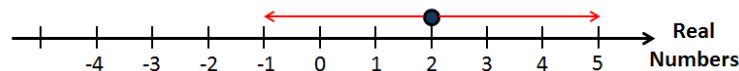
Student Activity

7 8 9 10 11 12



Introduction

What numbers are 3 units away from the number 2 on the real number line?



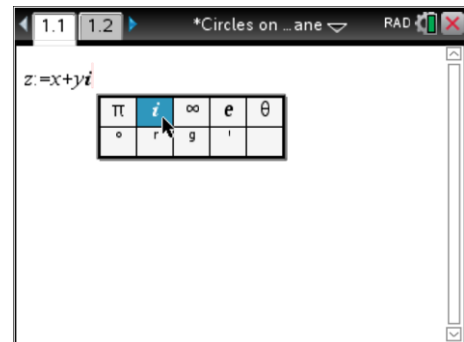
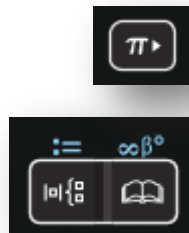
We can write this as $|x - 2| = 3$ where x represents the points on the real number line that are 3 units away from the number 2. What happens if the question is changed to include complex numbers? Complex numbers occur on a plane rather than a line. If z is used to represent the set of points that are 3 units away from the number 2, the equation is simply re-written as: $|z - 2| = 3$. What does this look like?

Understanding the Equation

Open the TI-nspire document: "Circles in the Complex Plane". The first page is a calculator application.

Define z as a complex number:

$$z := x + yi$$



The complex number i can be called up from the maths symbols and constants menu located on the π key. The multiplication sign between the y and i is implied and will automatically be inserted.

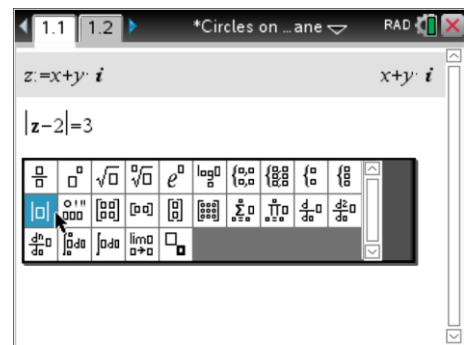
Use the mathematical operations template to locate the absolute value and enter the equation:

$$|z - 2| = 3$$

The equation will automatically be re-written in Cartesian form. Once the Cartesian equation is displayed, press the x^2 key followed by enter, both sides of the equation will be squared.

Compare the answer to the following:

$$(x - 2)^2 + y^2 = 9$$



Question: 1.

Describe the set of complex points that are 3 units from the point: $z = 2$.

Question: 2.

Write an equation for the set of points that would be 3 units from the point $z = 2i$.

Question: 3.

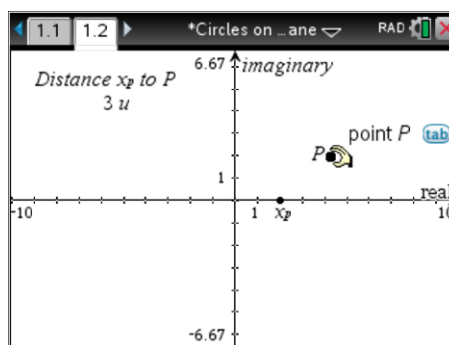
Write an equation for the set of point that would be 5 units from the point $3 + 4i$.

Geometric Exploration

Navigate to the Graph application on page 1.2. The point P has been set so that it will remain 3 units from the point x_p .

Move the mouse over point P and grab it by pressing CTRL + Click or by holding the mouse button down for approximately 2 seconds.

Drag point P around the Argand* plane. Notice that the point can only move along a fixed path.



Release point P by pressing **Esc**. Use the **menu** to access the **Trace** option followed by **Geometry Trace**. Click once on point P (this activates the trace feature) then click and grab point P and move it around once again.

Question: 4.

What path is traced out by Point P and what would be the corresponding equation?

The 'zeros' command finds all points in an expression that equal zero, it can be used to draw the set of points defined by:

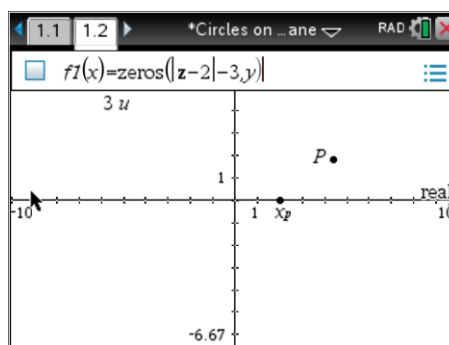
$$|z - 2| = 3$$

From the above rule it follows that:

$$|z - 2| - 3 = 0$$

In the equation entry line type:

$$\text{zeros}(|z - 2| - 3, y)$$

**Question: 5.**

Find the points that are 3 units from both: $z = 2 + 2i$ and $z = -2 - 2i$.

Question: 6.

A set of points z_n on the complex plane are located d units from the point: $z = a + bi$, two of the points are: $z_1 = 2i$ and $z_2 = -2$.

- If $d = 2$, determine two possible equations for the set of points.
- If $d = 10$, determine two possible equations for the set of points.
- Determine the relationship between a and b for any distance d .

