Teacher Notes

Activity 7

Graphs of Functions and Their Derivatives

Objectives

- See the derivative as an indicator of increasing/decreasing function behavior
- See the derivative as an indicator of local maxima/minima function behavior
- Graphically associate a function with its derivative

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 75 minutes

Abstract

In this activity, the concepts of increasing and decreasing function behavior are defined. This is followed by a graphical and symbolic exploration designed to show the student how and why the derivative can be used as an indicator for this behavior. The concepts of local maxima and minima are then informally defined, followed by questions that allow students to uncover the ideas behind the first derivative test. A series of questions invites students to synthesize these ideas by comparing graphs of functions and their derivatives. Finally, the derivative of a function f is compared to the derivative of $f + h$. This activity deals only with functions that are differentiable for all real numbers (except for one function in the matching exercise).

Management Tips and Hints

Prerequisites

Students should:

- be able to graph and generate tables of functions on the graphing handheld.
- know the definition of the derivative and have experience using derivatives.
- know the derivative of the sine function.

Note: *This activity would make a good precursor to the first derivative test.*

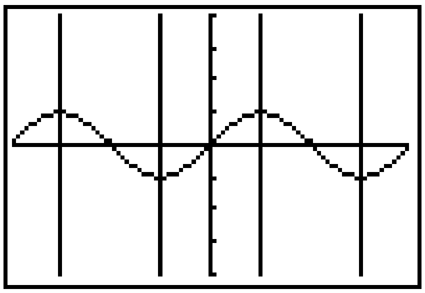
Evidence of Learning

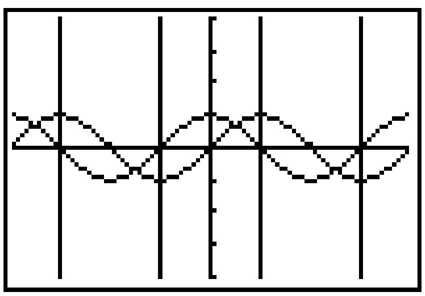
Students should be able to state how the derivative is an indicator of the original function's increasing or decreasing behavior and how the derivative can help find local maxima and minima where differentiable.

Extensions

After doing this activity, students should be challenged to draw a reasonably accurate graph of the derivative of a function given only the graphical representation of the function. One way to do this is to project a function directly onto a chalkboard using a ViewScreen™ and have a student draw the derivative graph directly on the board. The graphing handheld can then generate the graph of the derivative as a check.

Activity Solutions

1. 

2. 

3. When a function is increasing, the derivative is positive. When a function is decreasing, the derivative is negative.

4. Positive; $f(x + h) > f(x)$ because $(x + h) > x$ (h is positive) and f is increasing.

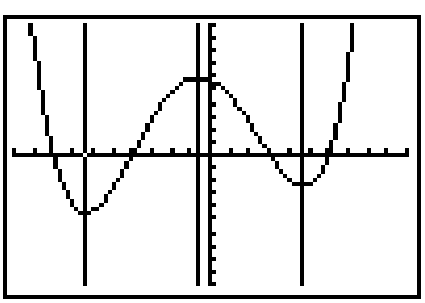
5. Positive; $f(x + h) - f(x) > 0$ and $h > 0$.

6. Negative; $f(x + h) < f(x)$ because $(x + h) < x$ (h is negative) and f is increasing.

7. Positive; $f(x + h) - f(x) < 0$ and $h < 0$.

8. Positive; $f(x + h) - f(x) > 0$ and $h > 0$.

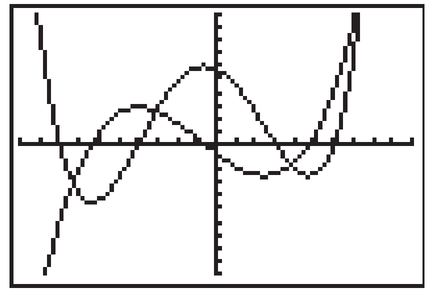
9. Positive; $f(x + h) - f(x) < 0$ and $h < 0$.

10. 

11. The derivative changes from positive to negative.

12. The derivative changes from negative to positive.

13. Yes



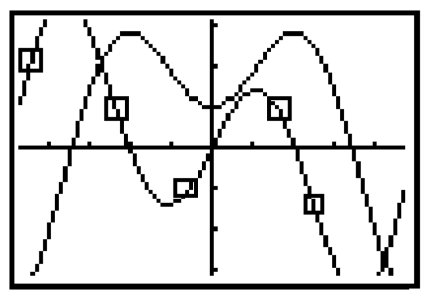
14. 0

15. It changes from negative to positive.

16. It changes from positive to negative.

17. The original function has a local minimum between $^{-}7$ and $^{-}6$ (looking at the graph) and a local maximum between $^{-}1$ and 0. So the results from Questions **15** and **16** are consistent with the answers to Questions **11** and **12**.

18. The graph of the derivative is identified with rectangles.



19. From top to bottom: *B*, *A*, *D*, *C*

20. They have the same shape.

21. The second function is shifted upward by two units.

22. The derivatives of these functions are identical because for any *x*, each function has the same tangent line slope.

$$\frac{d}{dx}f(x) = \frac{d}{dx}(f(x) + k)$$

Students may also know that the derivative of a sum is the sum of the derivatives and that the derivative of a constant function is zero. This type of symbolic reasoning should be connected to the graphical argument and vice versa.