Extraneous Solutions
Student Activity

Name _____

Class _____

In this activity, you will investigate real and extraneous solutions for radical equations conceptually, graphically, and algebraically. Special consideration should be given to the situations that create extraneous roots and why such solutions are a possibility.

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1. Solve the radical equation $\sqrt{2-x} = x$ algebraically and record your solutions.

   a. How many solutions did you find for the equation? Check algebraically to determine whether these solutions are correct.

   b. Visualize each side of the equation as separate functions. What does the graph of $f(x) = \sqrt{2-x}$ look like? What does the graph of $f(x) = x$ look like? In how many points might these two graphs intersect? How does this compare to your answer in part 1a?

Press [on] **> New Document > Add Graphs**.

2. In the entry line, enter the function $f1(x) = 2\sqrt{x-1}$ and press [enter]. Then press [ctrl] [G] to enter another function: $f2(x) = x - 1$.

   a. How many points of intersection are there for these two functions?

   b. What do you know about the points of intersection for these two functions?

3. To find the point(s) of intersection, select **Menu > Geometry > Points & Lines > Intersection Point(s).** Move the cursor on the handheld to both graphs and press [enter] on each graph. Where are the points of intersection?

4. An algebraic solution is necessary to prove that the intersection points are actually solutions to the equation. To solve the equation algebraically and check your solutions, add a new *Calculator* page to the document by selecting ctrl doc. Check whether the points of intersection are true solutions to the equation by substituting each *x*-value into the equation $2\sqrt{x-1} = x-1$. and enter the equation into the handheld. For example, check whether (5, 4) is a point of intersection. Enter a similar equation to check the other point of intersection.

   Did the truth value (either true or false) confirm your algebraic solution and the solution obtained by using the intersection tool? Explain.

5. Let's revisit the equation from question 1, $\sqrt{2-x} = x$. Check your solutions as you did in question 4 by adding a new *Calculator* page to the document and entering the equation. Remember that *true* implies the solution is correct, while *false* implies the solution is incorrect. Did the truth value confirm your solution algebraically and the solution obtained graphically? Explain.

6. To find the graphical solution of this equation, graph each side of the equation as a separate function. Add a new *Graphs* page to the document. Then enter $f3(x) = \sqrt{2-x}$. Press ctrl G to bring back the entry line to enter $f4(x) = x$. To find the point(s) of intersection, press **Menu > Geometry > Points & Lines > Intersection Point(s).** Select each graph and all intersections points will show. Record the point(s) of intersection.

   a. What do the point(s) of intersection of the two functions represent and where do they occur? Explain.

   b. How does the solution for this equation differ from the equation in question 2? Explain your reasoning.

   c. Explain why there are not two solutions, as in question 2.

d. Describe why and when the solutions of a radical equation occur and need to be checked.

7. It is important to check the solutions to equations containing a radical because some may be extraneous solutions.

   a. What is an extraneous solution of an equation?

   b. Why do extraneous solutions sometimes occur in the process of solving rational or radical equations? Explain your reasoning.