Tetrahedral Numbers

Student Worksheet

7   8   9   10   11   **12**

TI-30XPlus MathPrint™   Activity   Student   50 min

Finding Patterns

What are the Tetrahedral numbers? The prefix 'tetra' refers to the quantity four, so it is not surprising that a tetrahedron consists of four faces, each face is a triangle. This triangular formation can sometimes be found in stacks of objects. The series of diagrams below shows the progression from one layer to the next for a stack of spheres.

Row Number	1	2	3	4	5
Items Added					
Complete Stack					

Question: 1.

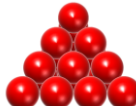Create a table of values for the row number and the corresponding quantity of items in a complete stack.
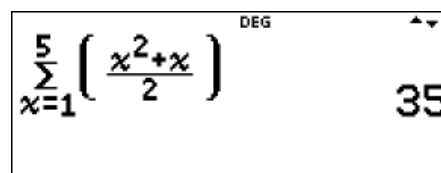
Question: 2.

Create a table of values for the row number and the corresponding quantity of items that are added to the stack.

Question: 3.

The calculator screen shown here illustrates how to determine the fifth tetrahedral number. The same command could be used to determine any of the tetrahedral numbers.
Explain how this command is working.
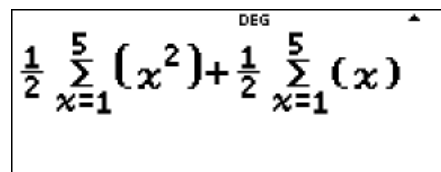
$$\sum_{x=1}^{5} \left(\frac{x^2+x}{2} \right) \qquad 35$$

Question: 4.

Verify that the calculation shown opposite is the same as the one generated in Question 3.

$$\frac{1}{2} \sum_{x=1}^{5} (x^2) + \frac{1}{2} \sum_{x=1}^{5} (x)$$

Question: 5.

Enter the numbers 1, 2 … 10 in List 1 on the calculator. Enter the first 10 tetrahedral numbers in List 2. Once the values have been entered try the following:

a)   Quadratic regression using List 1 and List 2. Check the validity of the result via substitution.

b)   Cubic regression using List 1 and List 2. Check the validity of the result via substitution.

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TEXAS INSTRUMENTS

Pascal's Triangle – Hidden Gem

Pascal's triangle also contains the tetrahedral numbers.

Notice: The nth triangular number is in row[1] $n+2$.

Example: The number 20 is the 4th triangular number and it is located in the 6th row.

Recall that the elements in Pascal's triangle can be computed using combinatorics: $^{n}C_{r} = \dfrac{n!}{(n-r)!\,r!}$

$$\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 \\
\end{array}$$

$$1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$$

Question: 6.

Use combinatorics to determine the value of the 100th Tetrahedral number. Check your answer using the cubic equation established in Question 5 and the summation tool on the calculator.

Question: 7.

Use Pascal's triangle to determine a formula for the Tetrahedral numbers.

Question: 8.

Given that the Tetrahedral numbers can be computed using:

$$\sum_{n=1}^{x}\left(\frac{n^{2}+n}{2}\right)$$

Use induction to show that this is equal to $\dfrac{x(x+1)(x+2)}{6}$.

Question: 9.

Question 4 used the property that $\displaystyle\sum_{n=1}^{x}\frac{n^{2}+n}{2}=\frac{1}{2}\sum_{n=1}^{x}n^{2}+\frac{1}{2}\sum_{n=1}^{x}n$.

Use this to show that: $\displaystyle\sum_{n=1}^{x}n^{2}=\frac{x(x+1)(x+2)}{6}$

Question: 10.

Use induction to prove that: $\displaystyle\sum_{n=1}^{x}n^{2}=\frac{x(x+1)(2x+1)}{6}$.

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TEXAS INSTRUMENTS