Loan Repayments

Teacher Notes and Answers

7 8 9 10 11 **12**

TI-30XPlus MathPrint™ | Investigation | Student | 30 min

Introduction

Teacher Notes: As teachers we see this topic as relevant to almost every student. For many teenagers the concept of owning a home and paying down the mortage is a long way off their radar. To increase student engagement, and as a follow up to this activity, students can use a website such as http://realestate.com.au to search for a house and then simulate the corresponding home loan repayments.

Example Calculations

One of the most significant financial commitments that most people make is to purchase a property. The vast majority of home buyers need to borrow a signficant amount of money. Minor changes in payments, payment methods and interest calculations can save many thousands of dollars over the course of the loan. Taking the time to understand how home loans work is therefore a great investment in itself.

Problem Statement

Jim decides to purchase a house, he needs to borrow $350,000 (Principal amount) from the bank. The annual interest rate is offered by the bank is 3% per annum calculated monthly. Monthly repayments of $2,000 are made at the end of each month.

Question: 1.

How much interest will Jim pay at the end of the first month? **Answer**: $350,000 x 0.03 / 12 = $875.00

Question: 2.

Based on your answer to Question 1 and the repayments stated in the problem:

i) How much money will Jim pay off the principal value[1] in the first month? [Effective Payment]
Answer: $2,000 – $875 = $1125

ii) How much money will Jim owe the bank at the end of the first month?
Answer: $350,000 + $875 - $2,000 = $348875 OR $350,000 – $1,125 = $348,875

iii) What percentage of Jim's first month's home loan repayment is reducing the principal amount?
Answer: 1,125 ÷ 2,000 = 56.25%

Calculator Instructions

The TI-30XPlus Multiview calculator includes active lists. If the lists contain any data it is best to clear them first.

Press: **[data]** twice and select option 4 – Clear all lists.

[1] Principal Value = Amount of money originally borrowed.

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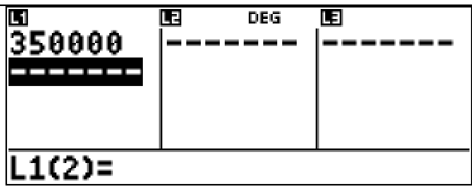
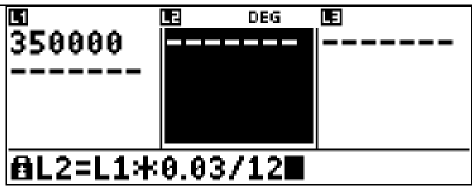
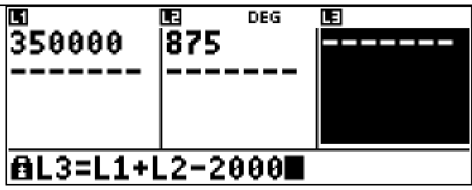
List 1 (L_1) will be used as the monthly opening balance. For the very first month in Jim's loan, this is equal to $350,000. Immediately after clearing the lists the calculator will return to the editor. If the editor is not visible; Press **[data]** and enter 350000 in L_1.	
List 2 (L_2) will hold the computed interest for the month. This will be done using a formula. Navigate across to List 2 and press **[data]** and navigate to Formula. Select option 1: Add / Edit Formula	
The formula for list 2 is relatively straight forward; it will calculate the interest for the month. Press the **[data]** key to access the List 1 variable. The remainder of the formula represents 3% interest (0.03) computed for 1/12th of a year (one month). Pressing **[enter]** executes the calculation.	
The next step is to enter a formula in List 3. Use the steps above to guide you to enter a formula in List 3. Formula: L1 + L2 – 2000 Note that the formula represents the 'opening balance' in List 1 ($350,000) plus the interest for the month in List 2 minus the monthly repayment ($2000)	

TIP!

Check the interest and monthly balance results with those obtained in Questions 1 & 2 before proceeding with the following questions.

Question: 3.

Navigate to the second line in List 1. Use the balance calculated for the end of the first month as the opening balance for the second month. [Type the value carefully!]

i) How much interest is Jim charged in the second month? Explain why this is different than the first month.
Answer: $872.19 It is different because he now has slightly less debt than before.

ii) How much money will Jim pay off the principal value in the second month? [Effective Payment]
Answer: $2,000 – $872.19 = $1127.81

iii) How much money will Jim owe the bank at the end of the second month?
Answer: $348875 + $872.19 - $2,000 = $347747.2 OR $348,875– $1127.81 = $347747.2

Question: 4.

Navigate to the third line in List 1. Use the balance calculated for the end of the second month as the opening balance for the third month. [Type the value carefully!]

i) How much interest is Jim charged in the third month?
Answer: $869.37

ii) How much money will Jim pay off the principal value in the second month?
Answer: $2,000 – $869.37 = $1130.63

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Question: 5.

Continue editing the opening balance, copy and complete the table for the first 12 months of the loan.

Month	1	2	3	4	5	6
Opening Balance	$350,000	$348,875.00	$347,747.20	$346,616.56	$345,483.10	$344,346.80
Interest Charged	$875.00	$872.19	$869.37	$866.54	$863.71	$860.87
Effective Payment	$1125.00	$1127.81	$ 1130.63	$1,133.45	$1136.29	$1139.13
Closing Balance	$348,875.00	$347,747.20	$346,616.56	$345,483.10	$344,346.80	$343,207.67

Month	7	8	9	10	11	12
Opening Balance	$343,207.67	$342,065.69	$340,920.86	$339,773.16	$338,622.59	$337,469.15
Interest	$858.02	$855.16	$852.30	$849.43	$846.56	$843.67
Effective Payment	$1,141.98	$1,144.84	$1,147.70	$1,150.57	$1,153.44	$1,156.33
Closing Balance	$342,065.69	$340,920.86	$339,773.16	$338,622.59	$337,469.15	$336,312.82

Question: 6.

Calculate the total amount of interest paid for the year and compare the amount to a flat rate of 3% interest.

Answer: $10,312.82. This amount can be calculated by summing List 2 on the calculator.
The purpose of the question is to highlight the difference between 12 x $875 = $10,500.00. The difference highlights the saving through the reduction in the principal value throughout the course of the year.

Using a Formula

The closing balance can be computed using a single formula:

$$Fv = Pv(1+r)^n - P\left(\frac{(1+r)^n - 1}{r}\right)$$ **- Equation 1**

Where:

- Fv = Final Value
- Pv = Principal Value
- P = Payment amount
- r = Interest rate (per payment)
- n = Number of payments

Question: 7.

The 12 month simulation generated by the lists and recorded in the table can be computed using Equation 1. Write down the values for Pv, P, r and n, hence determine the closing balance for the 12th month.
Check your answer against the one in your table.
Answer: $336,312.82 (formula) compared with $336,312.82 using the calculator lists.

TIP!

> There are a lot of variables in the Final Value formula so it is easy to make a simple error whilst attempting to evaluate the formula, particularly if you are doing multiple calculations. For this reason it is worthwhile defining the formula using the "Function" definition on the TI-30XPlus MathPrint.

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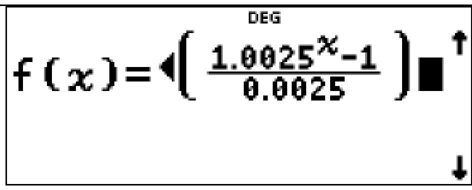
Calculator Instructions (Tip)

To define a rule or function on the calculator press the **[Table]** key and select option 1: Add / Edit Function.

The function allows for just one variable, in this case that will be the number of repayments (n), however the calculator requires the variable to be x , so the rule will be entered as:

$$f(x) = 350000 \times 1.0025^x - 2000\left(\frac{(1.0025)^x - 1}{0.0025}\right)$$

Where $1.0025 = 1 + \frac{3\%}{12}$



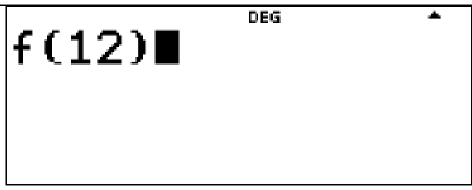
Comments:

- Right arrow to navigate out of exponent.
- Use the fraction key to insert the vinculum.
- Left arrow indicates content to the left.

Once the function has been defined, press **[enter]** then **[2nd] [mode]** to quit or exit.

To check that the function has been defined correctly, try calculating f(12).

Press [table] and select option 2, then enter 12 to calculate the balance at the end of the 12th month. The amount should be the same as that computed in the table.



Question: 8.

The loan can now be modelled for an extended period of time. Use the function defined on the calculator to determine the annual balance on the loan for 20 years. Copy and complete the table below.

Year	0	1	2	3	4	5	6
Opening Balance	$350,000	$336,312.82	$322,209.33	$307,676.87	$292,702.39	$277,272.45	$261,373.19
Interest Charged	$10,312.82	$9,896.51	$9,467.54	$9,025.52	$8,570.06	$8,100.74	$7,617.15

Year	7	8	9	10	11	12	13
Opening Balance	$244,990.34	$228,109.19	$210,714.58	$192,790.90	$174.322.06	$155,291.46	$135,682.04
Interest Charged	$7,118.85	$6,605.39	$6,076.32	$5,531.15	$4,969.41	$4,390.57	$3,794.13

Year	14	15	16	17	18	19	20
Opening Balance	$115,476.17	$94,655.73	$73,202.00	$51,095.75	$28,317.11	$4,845.63	0
Interest Charged	$3,179.55	$2,546.28	$1,893.74	$1,221.36	$528.53	$21.42*	

*Loan paid out in the third month using a reduced payment.

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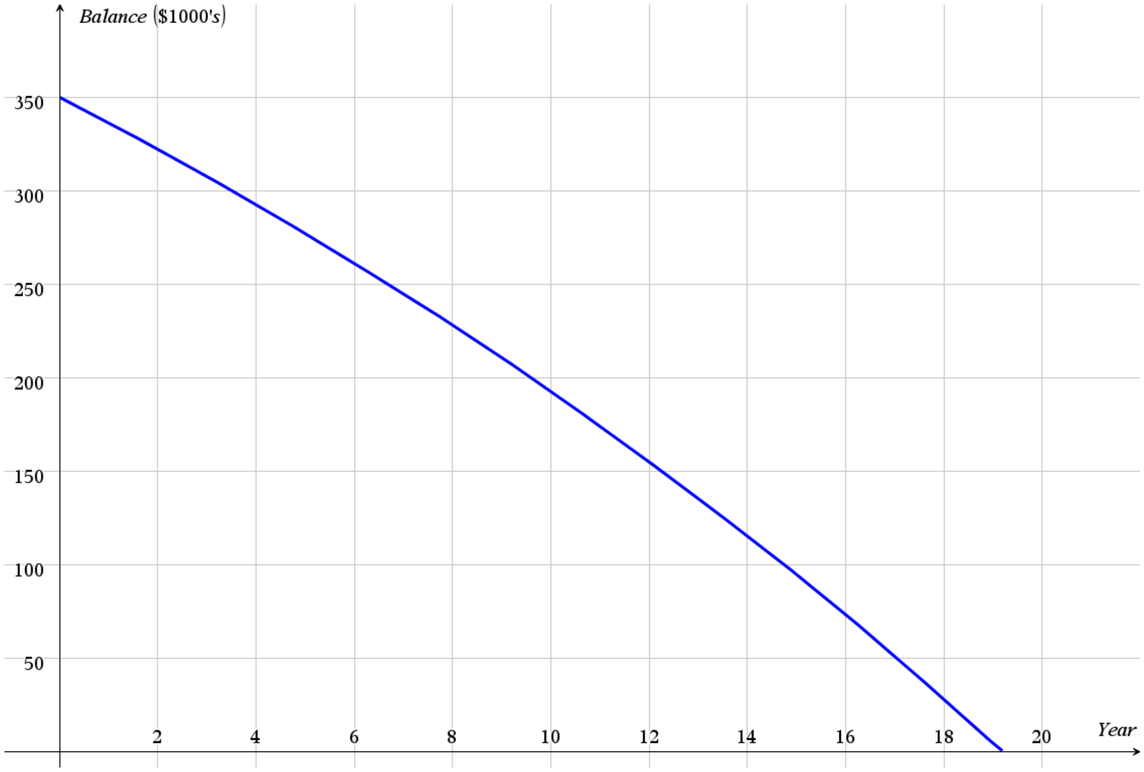
Question: 9.

Graph the annual balance of the loan over the course of the loan.



Question: 10.

Explain why the interest charged each year is decreasing.

Answer: As the balance on the loan decreases the amount of interest decreases also.

Question: 11.

Explain why the balance of the loan drops more rapidly throughout the course of the loan.

Answer: As the balance reduces, so too does the interest, therefore a greater and greater proportion of each payment contributes toward paying off the principal loan value.

Question: 12.

At the start of the second year of his mortgage, Jim receives a bonus of $29,500. Jim decides to pay this amount off his loan immediately. If Jim continues with his repayments as originally scheduled, explain why Jim will save almost $48,000 over the course of his loan as a result of this repayment.

Answer: The additional repayment saves interest over the duration of the loan. Another way to think of the lump sum payment is an immediate reduction in the loan balance, effectively transporting him into a point in the future that would have represented an equivalent loan balance.

The difference between the loan balances from year 2 to year 4 is $29,506. Jim's additional repayment therefore skips his loan ahead to this point. During this 2 year period Jim would have paid 2 years of repayments, equivalent to $48,000. So, even though the interest rate is relatively low (3%), the total saving over the course of the loan is much bigger than 3%!

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