Gateway Arc Length

GatewayArcLength.tns

Name ______________________

Class ______________________

Part 1 – Arc Length Introduced

The Gateway to the West is an inverted catenary arch in St. Louis that is about 630 feet tall and 630 feet wide at its base. A catenary (a hyperbolic cosine function) is the shape that a chain or cable forms when it hangs between two points. Page 1.4 gives a visual of this.

1. If you were to ride in the elevator tram of the Gateway Arch, you would travel at least how far to get to the top? Explain.

2. On page 1.8, use the formula to find the arc length from $x = 0$ to $x = 300$ for **f1**(x), the function which modeled the Gateway Arch. Write the formula and answer. Is this reasonable (when compared to your answer from Exercise 1)?

3. For the parametric equation $x(t) = 2\cos(t)$ and $y(t) = 2\sin(t)$, use the arc length formula to find the length from $t = 0$ to $t = \frac{\pi}{2}$. Show each step.

4. Use the *Calculator* application on page 2.5 to find the arc length of **f1**(x) = $\sqrt{4-x^2}$ from $x = 0$ to $x = 2$. Write out the equation and answer. Does this agree with the previous answer? Why or why not?

5. Approximate the arc length from $x = 0$ to $x = 3$ for $y = x^2 - 9$. Write the calculus formula and solution for this arc length. Try using **arcLen(f1(*x*),*x*,0,3)** on page 3.4 to check your answer.

6. Use the Pythagorean Theorem to approximate the arc length from $x = 0$ to $x = 3$ of $y = -x^2 + \frac{5}{3}x + 4$. Find the arc length. Write the formula and solution. Discuss if this is reasonable. Use the *Calculator* application on page 3.6 to check your answer.

Part 2 – Additional Practice

1. Which of the following integrals gives the length of the graph of $y = \arcsin \frac{x}{2}$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

a. $\int_a^b \sqrt{\frac{x^2+8}{x^2+4}}dx$ **b.** $\int_a^b \sqrt{\frac{x^2+6}{x^2+4}}dx$ **c.** $\int_a^b \sqrt{\frac{x^2-2}{x^2-4}}dx$

d. $\int_a^b \sqrt{\frac{x^2-5}{x^2-4}}dx$ **e.** $\int_a^b \sqrt{\frac{2x^2+3}{x^2+1}}dx$

2. The length of the curve determined by the parametric equations $x = \sin t$ and $y = t$ from $t = 0$ to $t = \pi$ is

a. $\int_0^\pi \sqrt{\cos^2 t+1}dt$ **b.** $\int_0^\pi \sqrt{\sin^2 t+1}dt$ **c.** $\int_0^\pi \sqrt{\cos t+1}dt$

d. $\int_0^\pi \sqrt{\sin t+1}dt$ **e.** $\int_0^\pi \sqrt{1-\cos t}\,dt$

3. Which of the following integrals gives the length of the graph of $y = \tan x$ between $x = a$ and $x = b$, where $0 < a < b < \frac{\pi}{2}$?

a. $\int_a^b \sqrt{x^2+\tan^2 x}\,dx$ **b.** $\int_a^b \sqrt{x+\tan x}\,dx$ **c.** $\int_a^b \sqrt{1+\sec^2 x}\,dx$

d. $\int_a^b \sqrt{1+\tan^2 x}\,dx$ **e.** $\int_a^b \sqrt{1+\sec^4 x}\,dx$