Side-Side-Angle: The Ambiguous Case

Math Objectives

- Students will identify the conditions necessary to determine a unique triangle when given two sides and a non-included angle (SSA).
- Students will justify why SSA is not sufficient to establish congruence of two triangles.
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

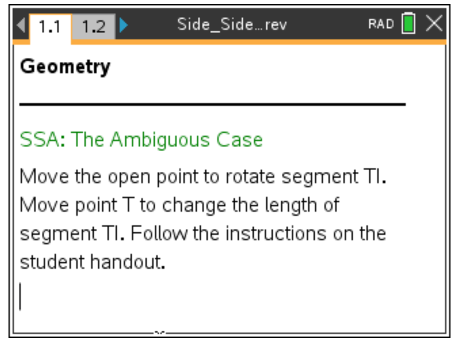
- triangle
- angle
- congruence
- Side-Angle-Side Congruence Theorem
- Angle-Angle-Side Congruence Theorem

About the Lesson

- This lesson involves manipulating the length of a side of a triangle and the non-included angle to identify the conditions that are necessary to determine a unique triangle.
- As a result, students will:
  - Be given two sides and the non-included angle, and they will change the size of the given angle to determine the conditions necessary to determine a triangle.
  - Conclude that uniqueness is a necessary condition for the congruence of two triangles.

TI-Nspire™ Navigator™ System

- Use Screen Capture to view possible triangles.
- Use Quick Poll to collect students' responses.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:
Student Activity
Side_Side_Angle_Student.pdf
Side_Side_Angle_Student.doc

TI-Nspire document
Side_Side_Angle.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

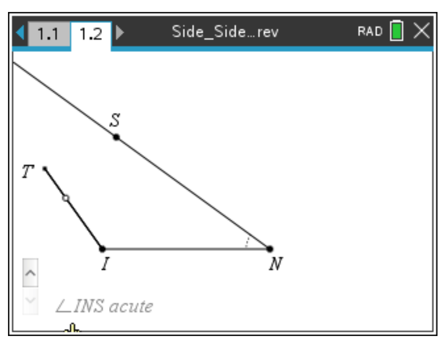
Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand (☝) getting ready to grab the point. Press ctrl ☝ to grab the point and close the hand (✊).

Be PATIENT when moving the points! It may take the handheld some time to process the move.

Teacher Tip: You might want to conduct the lesson as a teacher-led lesson using TI-Nspire™ Teacher Software because of the slow movement of the segments when you grab the open circle using the handheld. The file is optimized for use on the software. If TI-Nspire™ Student Software is available to your students, consider assigning the activity as an exploration the night before you plan to discuss the activity in class. When using the software, view the document in Computer mode for added color.

If students are using the handheld, you may want to demonstrate how to move the open circle to form a triangle in question 1b to make sure that students understand that only one triangle is formed and a message will appear on their screen when this happens. Because the pixels are so small, it appears as if many triangles are formed as $\overline{TI}$ gets closer to $\overrightarrow{NS}$. It is important that this be made clear to students rather than leaving them with a misconception.

Move to page 1.2.



1. Grab the open circle and rotate $\overline{TI}$.

   a. What changes? What remains the same?

   <u>Answer:</u> $\overline{TI}$ stays the same length. Angle N, $\overline{IN}$, and $\overrightarrow{NS}$ all stay the same. Angle I changes. It gets larger and smaller.

b. Grab the open circle to rotate $\overline{TI}$ until a message appears that a triangle has been formed. How many triangles are formed when point *T* is on $\overrightarrow{NS}$?

Answer: Exactly one right triangle is formed.

> **Tech Tip:** $\overline{TI}$ is long enough to form exactly one right triangle, with the vertex of the right angle at point *S*. Be sure that students note this triangle is a right triangle. Students may think a triangle has been formed before the message actually appears. Be sure they keep moving the open circle until they see the message. The action may be slow. Encourage students to be patient.

2. Move the open circle so that point *T* is not on $\overrightarrow{NS}$. Grab point *T* and drag it to make $\overline{TI}$ longer and shorter. Use the open circle to rotate $\overline{TI}$.

   a. What changes? What remains the same?

Answer: Whether $\overline{TI}$ becomes longer or shorter, Angle *N*, $\overline{IN}$, and $\overrightarrow{NS}$ all stay the same. Angle *I* changes. It gets larger and smaller.

> **Teacher Tip:** Remind students to move point *T* off of $\overrightarrow{NS}$ by using the open circle before trying to grab point *T*.

   b. How many triangles can you form?

   Answer: When $\overline{TI}$ is longer than when it started, it is possible to form one or two triangles. When $\overline{TI}$ is shorter, no triangles can be formed.

> **Teacher Tip:** When students have made $\overline{TI}$ equal to $\overline{IN}$, they may see only one triangle formed.

3. Fill in the following tables given the relationship of $\overline{TI}$ to $\overline{IN}$.

When $\angle INS$ Is Acute	
Length Relationship of $\overline{TI}$ to $\overline{IN}$	**Number of Triangles**
TI = *IN*	1
TI > *IN*	1
TI < *IN*	2, 1, or 0

Teacher Tip: Students need to explore with various lengths of $\overline{TI}$ when it is shorter than $\overline{IN}$ to ensure that they find the different triangles. Students can determine the relationship between segments $\overline{TI}$ and $\overline{IN}$ when $\overline{TI}$ is placed on top of $\overline{IN}$.

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 1 at the end of this lesson.

Click ∆ on the screen twice to change the size of $\angle INS$ to obtuse.

When $\angle INS$ Is Obtuse	
Length Relationship of $\overline{TI}$ to $\overline{IN}$	**Number of Triangles**
$TI = IN$	0
$TI > IN$	1
$TI < IN$	0

Click ∇ on the screen to change the size of $\angle INS$ to right.

When $\angle INS$ Is Right	
Length Relationship of $\overline{TI}$ to $\overline{IN}$	**Number of Possible Triangles**
$TI = IN$	0
$TI > IN$	1
$TI < IN$	0

4. a. Compare the three tables. What relationship between $\overline{TI}$ and $\overline{IN}$ will give you exactly one triangle?

Answer: When $TI > IN$, it appears that one triangle is formed no matter the size of $\angle INS$.

Teacher Tip: Students might also state that one triangle is formed when $TI = IN$ and $\angle INS$ is acute. Be sure students recognize that the only sure condition for determining a unique triangle is when the length of segment *TI* is greater than the length of segment *IN*.

b. State a general rule for being able to form exactly one triangle given any two sides and a non-included angle.

Answer: When the side that is not adjacent to the given angle is longer than the side adjacent to the given angle, you will be able to form one triangle.

Teacher Tip: You might test students' understanding by asking them to use their generalization to tell what would have to be true when given side *AB* and side *BC* and angle *C*.

5. a. Given two segments and the angle formed between them (SAS), how many triangles can you build? Explain your thinking.

   Answer: You can build only one triangle from two given segments and the included angle. Students may refer to a congruence statement to support their reasoning or they may argue that given the length of the segments and the width between them fixed (from the angle), there is only one length that will connect the endpoints of the two segments.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

   b. Given two segments and an angle not included between them (SSA), how many triangles can you build? Explain your thinking.

   Answer: As illustrated in the .tns file and the questions above for this activity, given parts corresponding to SSA you can get one, two, or no triangles.

6. Sherri and Linda were given the information below. Will the triangles they each created using this information always be congruent? Why or why not?

   a. The measure of $\overline{AB}$ was 6 inches. The measure of $\overline{BC}$ was 7.5 inches. The measure of $\angle B$ was 45°.

   Answer: The triangles will be congruent because the given information indicates that two sides and the included angle are congruent, which establishes SAS, a congruence theorem.

   b. The measure of $\overline{AB}$ was 6 inches. The measure of $\overline{BC}$ was 7.5 inches. The measure of $\angle C$ was 45°.

   Answer: Without drawing the figure, students will not be able to tell whether they can construct a triangle, as the side not adjacent to the given angle is shorter than the other given side. They might not be able to get a triangle at all if the side opposite the given angle is too short.

Teacher Tip: In fact, these given parts will not make a triangle, but students would have to actually try to construct one using the information to make sure. If they have constructed angles and segments of a given length earlier, they could try to construct this triangle to reinforce their reasoning.

c. The measure of $\overline{AB}$ was 7.7 inches. The measure of $\overline{BC}$ was 6 inches. The measure of $\angle C$ was 45°.

Answer: The triangles will be congruent, as the given side that is not adjacent to the given angle is longer than the side that is adjacent to the given angle. Thus, the given information determines a unique triangle.

d. The measure of $\overline{AB}$ was 6 inches. The measure of $\angle B$ was 45°. The measure of $\angle A$ was 52°.

Answer: The triangles will be congruent because the given information is Angle-Side-Angle (ASA), which is a congruence theorem.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

7. Why can SAS be used to prove triangles congruent but SSA cannot?

Sample Answer: Knowing two triangles have two corresponding adjacent sides and a corresponding non-included angle does not ensure the triangles will be congruent. This depends on the relationship of the lengths of the given sides with respect to the angle.

Depending on the measure of the non-included angle and the lengths of the sides opposite it and adjacent to it, it is possible to make one, two, or no triangles. Thus, even if the given parts are congruent, it might be possible to use these parts to create two different triangles as shown in the work above. Therefore, congruency cannot always be achieved from knowing that two sides and a non-included angle of one triangle are congruent to two sides and a non-included angle of another triangle.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- Given two sides and a non-included angle, it is not always possible to build a unique triangle.
- Given two sides of a triangle and a non-included angle, it is possible to build no, one, or two triangles, depending on the size of the given angle and the lengths of the given sides.
- SSA is not one of the triangle congruence theorems because it is possible to get no, one, or two triangles when given those parts of a triangle.

Assessment

Suppose you are given two sides and a non-included angle of a triangle. Sketch pictures to show when you will get two, one, or no triangles.

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Note 1

Question 3, *Screen Capture:* Use *Screen Capture* so that students can see a variety of possible triangles when $\angle INS$ is acute.

Note 2

Question 5, *Quick Poll:* Use an *Open Response Quick Poll* to collect students' responses to question 5. Have students share their thinking with the class.

Note 3

Question 6, *Quick Poll:* Use a *Yes/No Quick Poll* to collect students' responses to any parts of question 6.