Math Objectives

- Students will describe the relationship between a central angle, the radius, and the arc length of a circle.
- Students will recognize that changing the radius of a circle does not affect the proportions in the circle.
- Students will convert an angle measure from radians to degrees and vice versa.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- central angle
- radian
- exact value
- right angle

About the Lesson

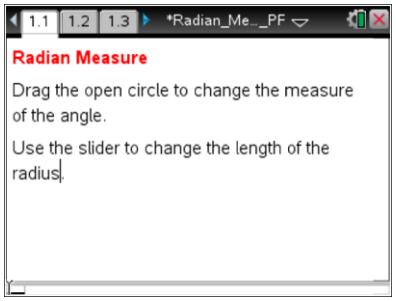
- This lesson involves exploring the relationship between the central angle, the arc, and the radius of a circle.
- As a result, students will
  - Use a slider to change the radius, and conclude that the relationships are preserved.
  - Form a definition of a radian.
  - Drag a point to change the measure of the central angle to discover the relationship between radian and degree measure.
  - Create a proportion that can be used to convert the measure of an angle from degrees to radians and vice versa.

TI-Nspire™ Navigator™ System

- Send out the *Radian_Measure.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Radian_Measure_Student.pdf
- Radian_Measure_Student.doc

TI-Nspire document

- Radian_Measure.tns

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Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand (☜) getting ready to grab the point. Also, be sure that the word *point* appears. Then select ctrl ☜ to grab the point and close the hand (✊). When finished moving the point, select esc to release the point. Select esc to hide the entry line if students accidentally select the chevron.

Tech Tip: Have the students tap the arrows on each slider to change the values of the slider.

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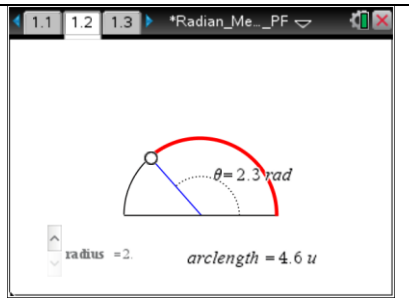

1. Drag the open circle until the arc length and the radius are equal. What do you observe about the radian measure of the central angle?

   Answer: The radian measure is 1 rad.

2. Drag the open circle farther along the arc.
   a. What is the central angle measure when the length of the arc is twice the length of the radius?

      Answer: The radian measure is 2 rad.

   b. What do you expect the arc length to be when the angle measure is 3 radians? Explain your reasoning.

      Answer: The arc length is 6 units based on a radius of 2 units.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

3. Select the slider to change the length of the radius. Are the observations you made in Questions 1 and 2 still true? Explain why or why not.

   Answer: Yes, when the angle measure is 1 rad, the arc length and radius are still equal. The arc length remains proportional to the radius.

4. Define a radian.

Answer: A radian is the measure of a central angle that spans an arc whose length is equal to the length of the radius.

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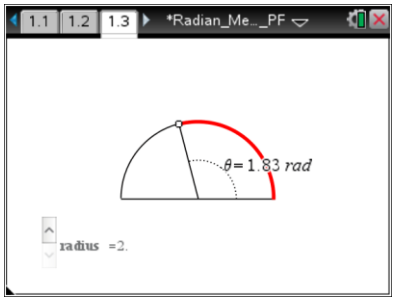

5. Drag the open circle counterclockwise as far as possible.
   a. What is the approximate radian measure of the central angle?

      Answer: The radian measure is approximately 3.14.

   b. What symbol do we use for this approximation?

      Answer: The angle measure is exactly π.

   c. What is the degree measure of the central angle?

      Answer: The measure of a straight angle is 180 degrees.

   d. Write an equation to represent the relationship between the radian and degree measures of the central angle.

      Answer: π radians = 180 degrees

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

6. Drag the open circle until the central angle is a right angle.
   a. Write this approximation as an exact value.

      Answer: The radian measure is approximately 1.57, which is exactly $\frac{\pi}{2}$.

   b. Write an equation to represent the relationship between the radian and degree measures of the right angle.

      Answer: $\frac{\pi}{2}$ radians = 90 degrees

7. Select the slider to change the radius. Do the relationships you discovered in Questions 5 and 6 remain the same? Why or why not?

   Answer: Yes, the relationships remain the same because the measure of the central angle never changes when only the radius varies.

8. How could you determine the exact radian measure of a 45-degree angle?

   Sample Answers: Answers will vary. Students might use the exact radian measure for 90 degrees and divide by 2 or use the exact radian measure for 180 degrees and divide by 4. They also might use a proportion. The exact radian measure of 45 degrees is $\frac{\pi}{4}$.

9. How could you determine the degree measure of an angle that measures $\frac{7\pi}{12}$ radians?

   Sample Answers: Answers will vary. Students might just replace the π radians with 180 degrees and simplify, or they might use a proportion. The angle measure is 105 degrees.

10. Write a proportion that can be used for converting any radian measure to degree measure and vice versa.

    Answer: $\frac{\pi}{180} = \frac{\text{radians}}{\text{degrees}}$

11. Use the proportion from Question 10 to determine the radian measure of a 280-degree angle.

    Answer: $\frac{\pi}{180} = \frac{x}{280}$; When solved, $x = \frac{14\pi}{9}$.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The relationship between a central angle, the radius, and the arc length of a circle.
- Changing the radius of a circle does not affect the proportions in the circle.
- How to convert an angle measure from radians to degrees and vice versa.

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Note 1

Question 2, *Quick Poll*

Send an open response Quick Poll to students asking for their predictions. Discuss as a class which of the predictions submitted they think will be the actual outcome and why.

Note 2

Question 5 part b, *Quick Poll*

Send an open response Quick Poll to see if students recognize the value 3.14159 as the approximation for π. Take this opportunity to discuss other decimal approximations, such as $\frac{\pi}{4}$ and $\frac{\pi}{2}$.

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