Unit Circle - Sine

Student Investigation

7 8 9 **10** **11** 12

TI-Nspire | Investigation | Student | 50 min

Aim

The aim of this investigation is to use the unit circle to explore patterns and relationships for the sine ratio and connect these with the corresponding graph.

Equipment

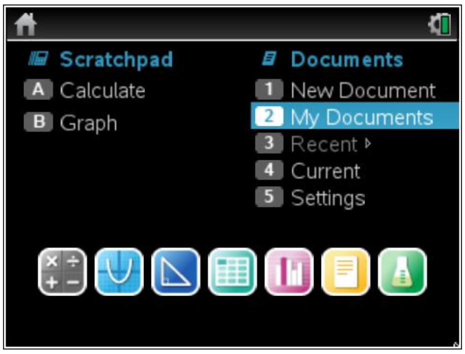
For this activity you will need:

- TI-nspire CAS
- TI-nspire CAS documents – Unit Sine

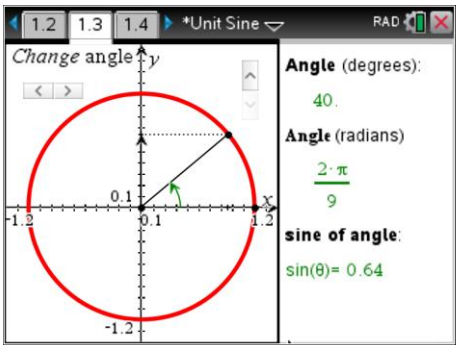
Introduction – Setting up the calculations

This activity requires access to the "Unit Sine" TI-nspire document.

Once the document is on your handheld, press the **home** key and select **My Documents**. Locate the "Unit Sine" document and press **Enter** to open.



Navigate to page 1.3. There are two applications displayed on this screen. On the left is a graph application that contains the unit circle, on the right is a notes application that displays the corresponding angles and the sine of the angle.



Place the mouse over the slider (top left) and click to increase or decrease the angle.

To animate the angle press **Ctrl** + **Menu** whilst the mouse is over the slider and select **Animate**. Repeat this process and select **Stop** to stop the animation.

Question: 1

Which axis is the sine ratio determined from in the unit circle? Y axis.

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Question: 2

Adjust or animate the slider and observe the value of the sine ratio. Identify the quadrants where the sine ratio is positive and where it is negative. Explain the results.

Note: You can display the quadrant numbers using the slider on the right of the unit circle.

The sine ratio is positive in the first and second quadrants as the y – axis is positive in both. The sine ratio is negative in the third and fourth as the y – axis is negative in both.

Question: 3

Adjust the angle to 30^{o} and record the sine ratio, then adjust the angle to 150^{o} and record the sine ratio.

$\sin(30^{o}) = \sin(150^{o}) = ½$

Question: 4

Adjust the angle to 45^{o} and record the sine ratio, then adjust the angle to 135^{o} and record the sine ratio.

$\sin(45^{o}) = \sin(135^{o}) = \frac{\sqrt{2}}{2} \approx 0.707$

Question: 5

Adjust the angle to 60^{o} and record the sine ratio, then find another angle that produces the same ratio.

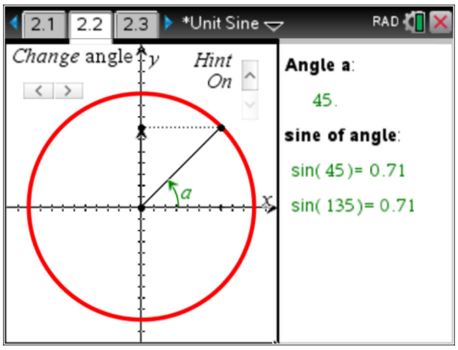
$\sin(60^{o}) = \sin(120^{o}) = \frac{\sqrt{3}}{2} \approx 0.866$

Question: 6

Adjust the angle to 70^{o} and record the sine ratio, then find another angle that produces the same ratio.

$\sin(70^{o}) = \sin(110^{o}) \approx 0.940$

Navigate to page 2.2. The unit circle has been reproduced on this page but with a focus on the angle relationship explored in the previous questions.



Change the angle displayed by using the slider in the top left corner of the unit circle. The angle can vary between -90^{o} and 90^{o}. The reflex angle is displayed for angles less than 0.

The slider in the top right corner of the unit circle can be used to display a 'clue' to help understand the relationship established in the previous questions.

Question: 7

Explore the relationship between the angles that produce the same sine ratio, explain the relationship geometrically. Use the 'hint' slider to help.

Angles measured counter-clockwise from the positive x axis are the same as angles measured clockwise from the negative x axis; similarly, angles measured clockwise from the positive x axis are equal to angles measured counter-clockwise from the negative x axis. Geometrically, the angles are a reflection in the y axis.

Question: 8

Page 2.3 contains a calculator application. Type the following expression: $\sin(\pi - x)$, write down the answer and explain the result.

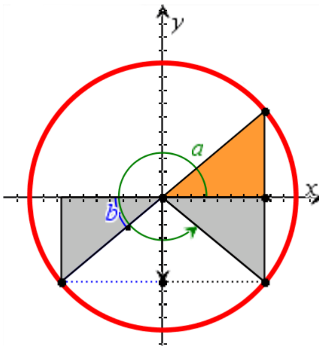
$\sin(\pi - x) = \sin(x)$. This is a symbolic way of representing the relationship identified in the previous question. $\sin(x)$ is measured counter-clockwise from the positive x axis and $\sin(\pi - x)$ is moving in the opposite direction $(-x)$ clockwise from the negative x axis (π).

Question: 9

In the calculator application, type the expression: $\sin(-x)$, write down the answer and explain the result. (Use diagrams to help explain the result)

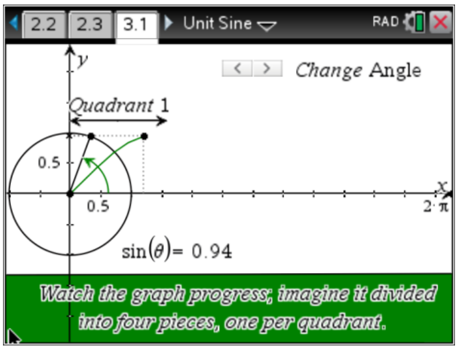
$\sin(-x) = -\sin(x)$.

For $\sin(-x)$ the angle is measured clockwise around the unit circle which is geometrically the same as reflecting the angle in the x axis. Sine is read from the y – axis, reflecting the y – axis in the x – axis swaps the sign: positive to negative or negative to positive. The diagram shows that reflecting the angle in the x axis produces a negative of the previous result.



Navigate to page 3.1. The unit circle on this page includes the graph of the sine ratio plotted against the angle.

Animate the angle and focus on the relationship between the unit circle and the graph. Imagine the sine graph split into four pieces, one piece per quadrant. Notice how the sine ratio repeats or becomes negative. Think of angles in the first quadrant as '**reference**' angles that can be used to compute the sine ratio anywhere else in the unit circle.



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Exact Angles

Angle (degrees)	0^{o}	30^{o}	45^{o}	60^{o}	90^{o}
Angle (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Question: 10

Without using a calculator determine the exact value of each sine ratio. Calculations are required for the angle in degrees (shaded row) and radian (clear row). The equivalent ratio column uses the reference angle. The first two have been done for you.

Angle	Reference Angle	Ratio	Equivalent Ratio	Result
150^{o}	30^{o}	$\sin(150^{o})$	$\sin(30^{o})$	$\frac{1}{2}$
$\frac{5\pi}{6}$	$\frac{\pi}{6}$	$\sin\left(\frac{5\pi}{6}\right)$	$\sin\left(\frac{\pi}{6}\right)$	$\frac{1}{2}$
210^{o}	30^{o}	$\sin(150^{o})$	$-\sin(30^{o})$	$-\frac{1}{2}$
$\frac{7\pi}{6}$	$\frac{\pi}{6}$	$\sin\left(\frac{7\pi}{6}\right)$	$-\sin\left(\frac{\pi}{6}\right)$	$-\frac{1}{2}$
300^{o}	60^{o}	$\sin(300^{o})$	$-\sin(60^{o})$	$-\frac{\sqrt{3}}{2}$
$\frac{5\pi}{3}$	$\frac{\pi}{3}$	$\sin\left(\frac{5\pi}{3}\right)$	$-\sin\left(\frac{\pi}{3}\right)$	$-\frac{\sqrt{3}}{2}$
135^{o}	45^{o}	$\sin(135^{o})$	$\sin(45^{o})$	$\frac{\sqrt{2}}{2}$
$\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\sin\left(\frac{3\pi}{4}\right)$	$-\sin\left(\frac{\pi}{4}\right)$	$\frac{\sqrt{2}}{2}$
120^{o}	60^{o}	$\sin(120^{o})$	$\sin(60^{o})$	$\frac{\sqrt{3}}{2}$
$\frac{2\pi}{3}$	$\frac{\pi}{3}$	$\sin\left(\frac{2\pi}{3}\right)$	$\sin\left(\frac{\pi}{3}\right)$	$\frac{\sqrt{3}}{2}$
215^{o}	45^{o}	$\sin(215^{o})$	$-\sin(45^{o})$	$-\frac{\sqrt{2}}{2}$
$\frac{5\pi}{4}$	$\frac{\pi}{4}$	$\sin\left(\frac{5\pi}{4}\right)$	$-\sin\left(\frac{\pi}{4}\right)$	$-\frac{\sqrt{2}}{2}$

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